For STEM and legal professions, simulate robotic reasoning

Workspace and time

Use as much space and as much time as you need to reliably get the answer in a presented format that can be readily audited without embarrassment. Using less paper and time than you need to work with professional reliability wastes opportunities to do your best.

Organize work predominantly in vertical columns (**top to bottom**) with ample **whitespace**. There is never anything such as "this is messy because it is only scratchwork for now."

	Suggested workspace
Short arithmetic/algebra problems	A single sheet of paper
Hard-to-read word problems in physics	Half-sheets of paper and/or index cards scattered across a large table

Always write slowly enough for every symbol to be perfectly clear, even to people unfamiliar with your handwriting. There is no such thing as "I will slow down and be more careful on the test." Slowly applying logic during homework (perhaps taking over 20 minutes to figure out how to do a single problem) develops the ability to figure out how to do never-before-seen problems on tests.

Perception

Read a paragraph's worth of information (e.g. problem, lettered part of a problem, notes/work you have developed during the problem)

1. Read every word in passage out loud and then quietly wait 3 seconds.
2. Read every word in passage out loud and then quietly wait 3 seconds.
3. Read every word in passage out loud.

Line-by-line analysis (do this for each clause in the passage)

1. Read every word in clause out loud.
2. Ask, "Does this clause **contain any logical content that differs from** any logical content **already read** in the same passage**?**" To answer this question, you might very well need to re-read (many times) the clause currently under study and previously read sentences.
    a. If so, **highlight** the portion of the clause that contains the **new logical content**.
    b. Otherwise, do not add additional highlighting to the clause. Instead, move on to analyze the next clause.

Scan and narrate an illustration

Pointing with your finger on the page to stabilize your gaze, slowly raster scan a 5-character (foveal) field of view across an illustration. Periodically pause to narrate what you see.

Scan the problem

Pointing with your finger on the page to stabilize your gaze, slowly raster scan a 5-character (foveal) field of view across the problem statement.

Scan everything printed/written on the page so far

Pointing with your finger on the page to stabilize your gaze, slowly raster scan a 5-character foveal visual field across the entire page. Periodically pause to read/narrate what you see.

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Situational awareness

Highlight key vocabulary words, quantities, and directives in problem statement.

Re-**write** the **problem statement** out by hand (near top of workspace).

Draw (start near top of workspace if purpose of illustration is initial translation of given information).

1. Convert a **sentence** that is difficult to read into a grammatical **diagram** or **flowchart**. For example, "the city budget in 2010 was 5% more than the city budget in 2000," can be expressed using the following flowchart:

Budget 2010	was	5% of	Budget 2000	more than	Budget 2000
Budget 2010	=	(0.05)	Budget 2000	+	Budget 2000

2. **Illustrate the number of situations** (for example, an "initial" situation and a "final" situation, or, for example, logical contingencies) by using vertical lines to divide the page into columns, one for each situation. The space in an individual column is available for drawings and analysis for an individual situation, with results from one or more columns possibly combined later on to arrive at a final result.
3. **Illustrate each situation** of interest. For each situation, clearly illustrate the object of focal attention and relevant aspects of its surrounding environment. Use a dashed bubble to identify the system of interest, if applicable. Each illustration of a situation needs to occupy a region on the page measuring at least $8\ \mathrm{cm}$ in width and $8\ \mathrm{cm}$ in height.
4. **Label objects**.
5. **Label each quantity** in each illustrated situation **with a variable and**, as much as possible, with **a numerical value** and with **appropriate units**.

   When a **variable** is **used for the first time,** use an arrow and/or fragment of English to **declare** its **meaning** clearly.

   Use subscripts when needed **to prevent** accidentally **assuming that similar-looking variables** have the same value. See the convention box below.
6. Draw **arrow to label** quantity/variable in equation, expression, or sentence with caption words, small diagrams, and/or equations/relationships. See the convention box below.
7. Draw **coordinate axes** indicating positive directions and origin.
8. For physics problems, consider drawing a **free-body diagram** using the list of questions from the Newton's laws cribsheet to identify relevant forces.
9. When asked to illustrate **vector addition**, arrange added vectors so that they touch **head-to-tail**.
10. Add a directed **edge** or **node** in a **"leads to" flow diagram**.

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Translate English to math or math to English
See convention box below for example of way to use subscripts in mathematical expressions.

Convention for using subscripts to assist translation from English sentences to mathematical equations

Convention
English: "[Property p] of [Item n] is (phrase, expression, and/or quantity)."
Equation: $p_n =$ (expression/quantity)

Example
English: "The mass of the Earth is 5.98×10^{24} kg.
Equation: $M_{\mathrm{E}} = 5.98\times10^{24}$ kg.

Example mistake: Confusing a prepositional phrase with the noun it describes
English: "[Property p] of [Item n] is (expression/quantity)."
Incorrect equation: $n =$ (expression/quantity)

Example mistake: Incomplete sentence (forgetting variable name and equal sign)
English: "[Property p] of [Item n] is (expression/quantity)."
Incomplete label on diagram: (expression/quantity)

Make a table: Generic example

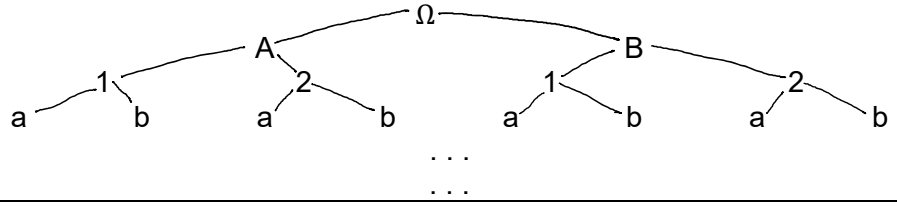
	Property A	Property B	. . .	Property Z
Item/Situation 1				
Item/Situation 2				
. . .				
Item/Situation N				

Make a table: Contingency cross

	Contingency A	Contingency B	. . .	Contingency Z
Contingency 1				
Contingency 2				
. . .				
Contingency N				

Make a tree branching diagram of contingencies

Universe
Categories A, B
Categories 1, 2
Categories a, b
. . .
Categories i, ii



Make a table: ICE/F chart

	Species A	Species B	Species C	⇌ | →	Species D	Species E	Species F
I							
C							
E/F							

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Make a table: Mixing problem

	Mixture 1	Mixture 2	Combined mixture
Amount of substance A			
Amount of substance B			
Volume or mass			

Make a table: Combined work-rate problems

	Rate	Time	Accomplished amount
Worker A			
Worker B			
Together			

Make a table: Kinematics

Horizontal	**Vertical**	**Trigonometry**
Initial t_i $=$		
x_i $=$	y_i $=$	
$v_{x,i}$ $=$	$v_{y,i}$ $=$	
Between initial and final times		
a_x $=$	a_y $=$	
Final t_f $=$		
x_f $=$	y_f $=$	
$v_{x,f}$ $=$	$v_{y,f}$ $=$	
Calculations		

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Givens (near top of workspace)

G:

Write down a list of given information not already included in a figure or a table, with each piece of information expressed as an equation or a complete sentence. See the convention box above.

Check units

Do the units for the quantity I just read make sense? Do I need to convert the units of a quantity before substituting that quantity into an equation?

Unit conversion

Particularly for conversion to base units, start ASAP (no need to wait until end of problem)

Given quantity ~~a~~	# ~~b~~	# ~~c~~	# ~~d~~	. . .
	# ~~a~~	# ~~b~~	# ~~c~~	. . .

Write the converted version of the quantity (in desirable units) near the original version (in undesirable units), and strike out the original version.

Requested unknowns (near top of workspace or bottom of workspace, depending on taste)

?:

Write down goal/conclusion/quantity to be determined.

Remind yourself of goal

Say, “At the end of the day, I would like to _____ (goal). Most immediately, I am trying to _____ (action). I have not yet accomplished this because _____ (difficulty). I might have misidentified the goal of the question, so I will keep my eyes open for information that might correct misconceptions I have about what I would like to find out.”

Check whether done

Ask, “Have I addressed the question/directive?” If so, stop.

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Retrieval

State a definition/defining diagram

1. State definition in sentence form:
2. Defining equation/relationship expressed using variables (letters, no numerical values substituted):

State a postulate/law/rule/theorem

1. Name of postulate/law/rule/theorem: _____

2. If idiomatic to the postulate/law/rule/theorem, express the law/rule/theorem using an equation expressed using variables (letters, no numerical values substituted). Otherwise, proceed to step 3.

3. Fill out this form if your postulate/law/rule/theorem has the form of an "if-then" statement.

Checklist of conditions	⇒	List of conclusions that are guaranteed when conditions are satisfied:

4. Fill out this form if your law/rule/theorem is a biconditional ("if and only if"):

Checklist of propositions	⇔	Checklist of propositions

5. Heuristic check for invalid postulate/law/rule/theorem: If you are not sure whether you have stated a previously introduced/derived postulate/law/rule/theorem, you should probably do a better job memorizing (or laying out on your presentation boards) your postulates, laws, rules, and theorems. Additionally, you should ask whether you can think of any exceptions (counterexamples) that would prove your alleged postulate/law/rule/theorem clearly false.

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Algebraic reasoning

Substitute
The **target equation** contains the quantity/variable to be replaced. The **source equation** provides an alternative expression for the quantity/variable to be replaced.

1. Use a dashed circle in the target equation to identify the quantity/variable to be replaced.
2. Use another dashed outline to circle the same quantity/variable in the source equation, where the quantity/variable is **isolated**.
3. In the source equation, use a solid outline to circle the side of the equation not already circled by a dashed outline.
4. Write a copy of the target equation, with the quantity/variable identified by a dashed circle replaced with parentheses containing the expression outlined by a solid circle in step 3.

Re-express
Show a single step that re-expresses one or both sides of a previous equation in a way that respects properties of arithmetic operations. Do not apply undefined operations (e.g. do not attempt to divide by zero).

Property of equality
Show the application of one same algebraic operation on both sides of an equation. Do not apply undefined operations (e.g. do not attempt to divide by zero).

CAUTION: Typically, the set of operations that can be performed on both sides of an inequality is smaller than the set of operations that can validly be performed on both sides of an inequality.

Declare a substitution variable
Let $u =$

Clean-up
For AP Calculus purposes, ensure that final expressions satisfy the following
1. No negative exponents
2. Combine like terms

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Vectorial reasoning

Graphical vector addition
Draw vectors to be added touching head-to-tail.

Make a table: Vector component chart
(Helpful for determining what to write in Newtonian laws like $a_x = \frac{\sum F_x}{m}$).

Vector	x-comp.	y-comp.
$\vec{\mathbf{A}}$	A_x	A_y
$\vec{\mathbf{B}}$	B_x	B_y
$\vec{\mathbf{A}} + \vec{\mathbf{B}}$	$A_x + B_x$	$A_y + B_y$

Resolve components
1. Identify the component of interest (e.g. x-component).
2. Does the vector have a component in the direction of interest? If so, is the component positive or negative?
3. Swipe finger across the coordinate axis corresponding to the component of interest.
4. Does my finger touch the arc labeling an angle (e.g. θ) of interest?

$$v_{\text{ADJ}} = v\cos\theta, \quad v_{\text{NOT-ADJ}} = v\sin\theta$$

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Determine whether the word “because” can be used to connect two independent clauses

Represent possibilities that have yet to be eliminated

Example format: List

Possibility 1: (Written description/diagram)
Possibility 2: (Written description/diagram)
Possibility 3: (Written description/diagram)
Possibility 4: (Written description/diagram)
⋮

Example format: Contingency table

		Possible values of variable L		
		A	B	⋯
Possible values of variable n	1			
	2			
	⋮			

State potentially useful requirement provided by problem or during course instruction

Requirement: (Written description/diagram)

Eliminate possibilities, if possible

- Announce out loud: “Possibility 1 is that [description].”
- Announce out loud: “The requirement is that [description].”
- Does this requirement contradict possibility 1? If so, cross out possibility 1.

- Announce out loud: “Possibility 2 is that [description].”
- Re-announce out loud: “The requirement is that [description].”
- Does this requirement contradict possibility 2? If so, cross out possibility 2.

⋮

- Announce out loud: “The final possibility is that [description].”
- Re-announce out loud: “The requirement is that [description].”
- Does this requirement contradict the final possibility? If so, cross out the final possibility.

Report result

Result	Example of phrase that can be used to report result
None of the possibilities were eliminated	[Requirement] did not eliminate any possibilities.
At least one possibility was eliminated	[Written description that precisely summarizes those possibilities that were not eliminated] because [requirement].

Two-column proof

Given:
To be shown:

Statement	Reason
1.	1.
2.	2.
3.	3.
⋮	⋮