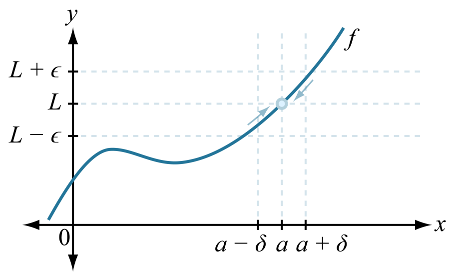
Limits formalize the notion of “approaching”

Finite limit at a particular value of x



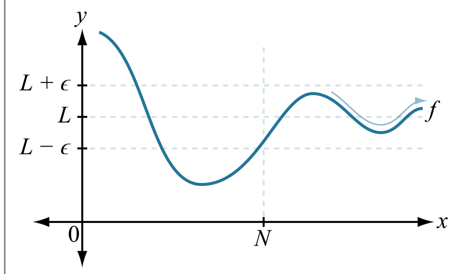
We write

$$\lim_{x \to a} f(x) = L$$

to informally mean that the value of $f(x)$ can be made to be arbitrarily close to L by requiring x to be sufficiently close to a while not equaling a

and to formally mean that so long as ϵ is a positive number, there exists a positive number δ so that trapping x in $(a-\delta, a) \cup (a, a+\delta)$ guarantees that $f(x)$ is trapped in $(L-\epsilon, L+\epsilon)$.

Finite limit “at” infinity



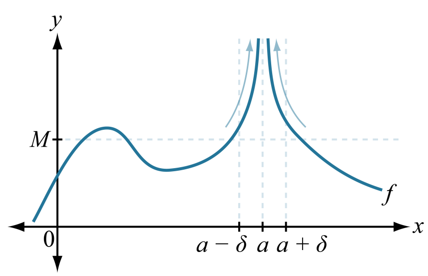
We write

$$\lim_{x \text{"}\to\text{"} +\infty} f(x) = L$$

to informally mean that the value of $f(x)$ can be made to be arbitrarily close to L by requiring x to be a sufficiently large positive number

and to formally mean that so long as ϵ is a positive number, there exists a number N so that trapping x in $(N, +\infty)$ guarantees that $f(x)$ is trapped in $(L-\epsilon, L+\epsilon)$.

Infinite limit at a particular value of x



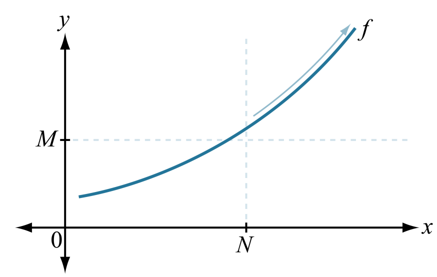
We write

$$\lim_{x \to a} f(x) \text{ "="} +\infty$$

to informally mean that the value of $f(x)$ can be made to be positive and arbitrarily large by requiring x to be sufficiently close to a while not equaling a

and to formally mean that so long as M is a number, there exists a positive number δ so that trapping x in $(a-\delta, a) \cup (a, a+\delta)$ guarantees that $f(x)$ is trapped in $(M, +\infty)$.

“Infinite limit” “at” infinity



We write

$$\lim_{x \text{"}\to\text{"} +\infty} f(x) \text{ "="} +\infty$$

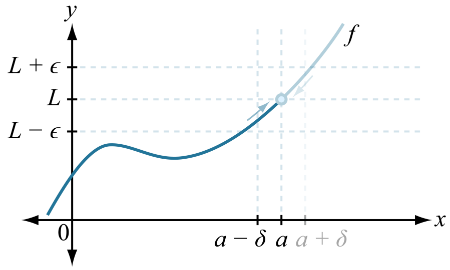
to informally mean that the value of $f(x)$ can be made to be positive and arbitrarily large by requiring x to be a sufficiently large positive number

and to formally mean that so long as M is a number, there exists a number N so that trapping x in $(N, +\infty)$ guarantees that $f(x)$ is trapped in $(M, +\infty)$.

Convention for naming values a, N, and M adopted from Stewart *Calculus* 4th ed.

Limits formalize the notion of “approaching”

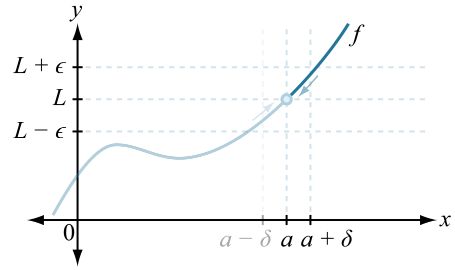
One sided-limit from the left



$$\lim_{x \to a^-} f(x) = L$$

The value of $f(x)$ can be made to be arbitrarily close to L by requiring x to be sufficiently close to a while $x < a$.

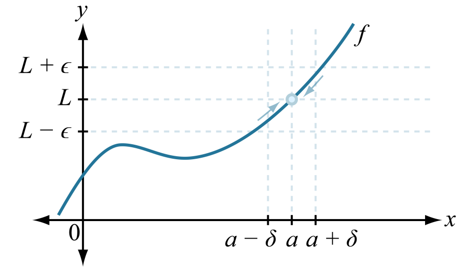
One-sided limit from the right



$$\lim_{x \to a^+} f(x) = L$$

The value of $f(x)$ can be made to be arbitrarily close to L by requiring x to be sufficiently close to a while $x > a$.

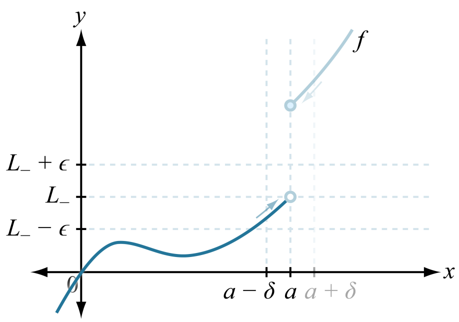
Limit



$$\lim_{x \to a} f(x) = L$$

The value of $f(x)$ can be made to be arbitrarily close to L by requiring x to be sufficiently close to a while not equaling a.

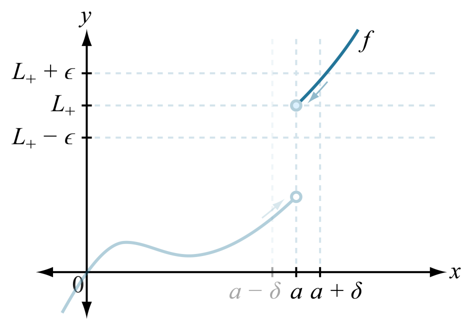
One-sided limit from the left



$$\lim_{x \to a^-} f(x) = L_-$$

The value of $f(x)$ can be made to be arbitrarily close to L_- by requiring x to be sufficiently close to a while $x < a$.

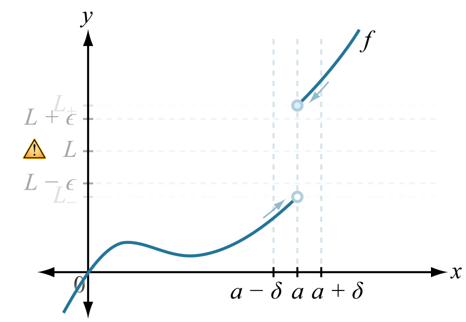
One-sided limit from the right



$$\lim_{x \to a^+} f(x) = L_+$$

The value of $f(x)$ can be made to be arbitrarily close to L_+ by requiring x to be sufficiently close to a while $x > a$.

Limit



$$\lim_{x \to a} f(x) \text{ DNE}$$

The value of $f(x)$ cannot be made to be arbitrarily close to a single value L by requiring x to be sufficiently close to a while not equaling a.