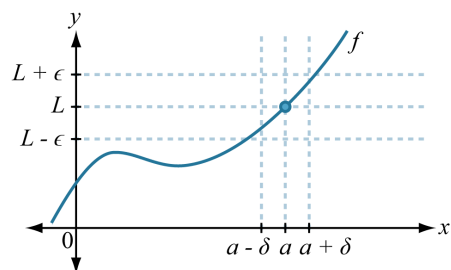


Limits formalize the notion of “approaching”

Finite limit at a particular value of x



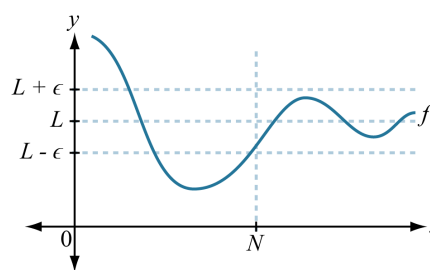
We write

$$\lim_{x \rightarrow a} f(x) = L$$

to informally mean that the value $f(x)$ can be made to be arbitrarily close to L by requiring x to be sufficiently close to a , but not equal to a

and to formally mean that so long as ϵ is a positive number, there exists a positive number δ so that trapping x in $(a - \delta, a) \cup (a, a + \delta)$ guarantees that $f(x)$ is trapped in $(L - \epsilon, L + \epsilon)$.

Finite limit “at” infinity



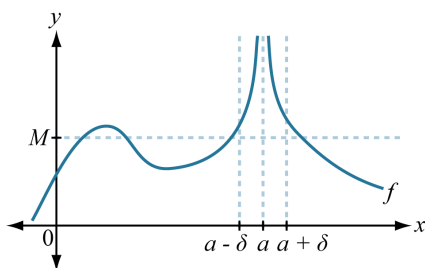
We write

$$\lim_{x \rightarrow +\infty} f(x) = L$$

to informally mean that the value $f(x)$ can be made to be arbitrarily close to L by requiring x to be sufficiently positively large

and to formally mean that so long as ϵ is a positive number, there exists a number N so that trapping x in $(N, +\infty)$ guarantees that $f(x)$ is trapped in $(L - \epsilon, L + \epsilon)$.

Infinite limit at a particular value of x



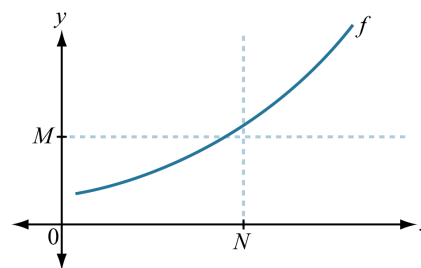
We write

$$\lim_{x \rightarrow a} f(x) = +\infty$$

to informally mean that the value $f(x)$ can be made to be arbitrarily positively large by requiring x to be sufficiently close to a , but not equal to a

and to formally mean that so long as M is a number, there exists a positive number δ so that trapping x in $(a - \delta, a) \cup (a, a + \delta)$ guarantees that $f(x)$ is trapped in $(M, +\infty)$.

“Infinite limit” “at” infinity



We write

$$\lim_{x \rightarrow +\infty} f(x) = +\infty$$

to informally mean that the value $f(x)$ can be made to be arbitrarily positively large by requiring x to be sufficiently positively large

and to formally mean that so long as M is a number, there exists a number N so that trapping x in $(N, +\infty)$ guarantees that $f(x)$ is trapped in $(M, +\infty)$.

Convention for naming values a , N , and M adopted from Stewart *Calculus* 4th ed.