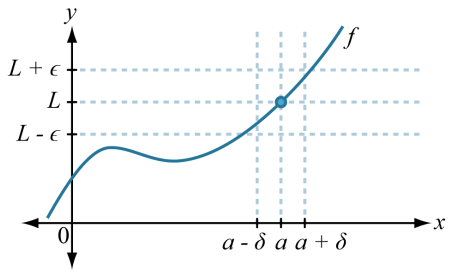
Limits formalize the notion of “approaching”

Finite limits at particular points



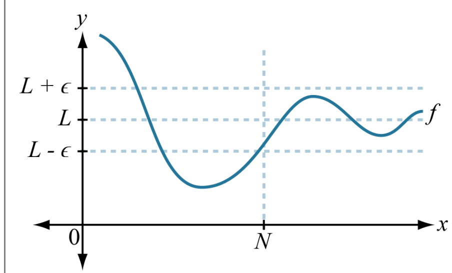
We write

$$\lim_{x \to a} f(x) = L$$

to informally mean that the value $f(x)$ can be made to be arbitrarily close to L by requiring x to be sufficiently close to a, but not equal to a

and to formally mean that so long as ϵ is a positive number, there exists a positive number δ so that trapping x in $(a-\delta, a) \cup (a, a+\delta)$ guarantees that $f(x)$ is trapped in $(L-\epsilon, L+\epsilon)$.

Finite limits “at” infinity



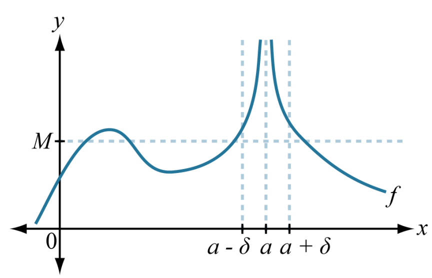
We write

$$\lim_{x \to +\infty} f(x) = L$$

to informally mean that the value $f(x)$ can be made to be arbitrarily close to L by requiring x to be sufficiently positively large

and to formally mean that so long as ϵ is a positive number, there exists a number N so that trapping x in $(N, +\infty)$ guarantees that $f(x)$ is trapped in $(L-\epsilon, L+\epsilon)$.

Infinite limits at particular points



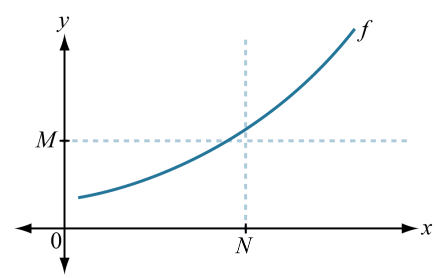
We write

$$\lim_{x \to a} f(x) \text{ "} = \text{" } +\infty$$

to informally mean that the value $f(x)$ can be made to be arbitrarily positively large by requiring x to be sufficiently close to a, but not equal to a

and to formally mean that so long as M is a number, there exists a positive number δ so that trapping x in $(a-\delta, a) \cup (a, a+\delta)$ guarantees that $f(x)$ is trapped in $(M, +\infty)$.

“Infinite limits” “at” infinity



We write

$$\lim_{x \to +\infty} f(x) \text{ "} = \text{" } +\infty$$

to informally mean that the value $f(x)$ can be made to be arbitrarily positively large by requiring x to be sufficiently positively large

and to formally mean that so long as M is a number, there exists a number N so that trapping x in $(N, +\infty)$ guarantees that $f(x)$ is trapped in $(M, +\infty)$.

Convention for naming values a, N, and M adopted from Stewart *Calculus* 4^{th} ed.