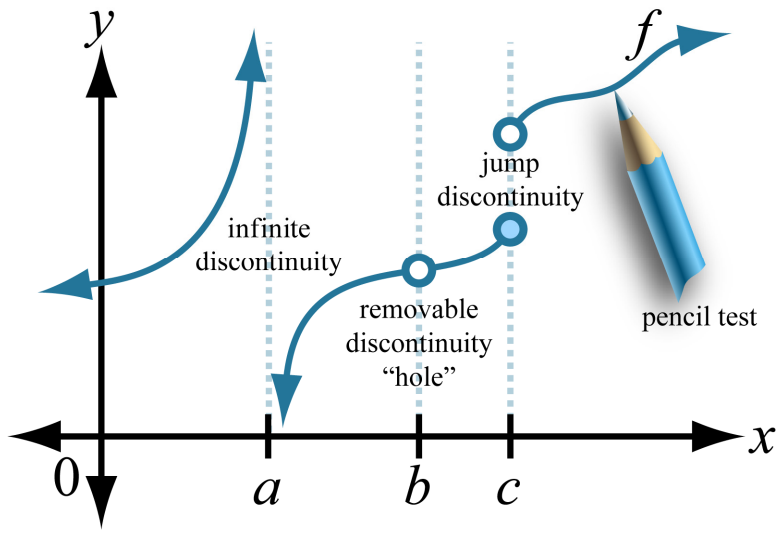
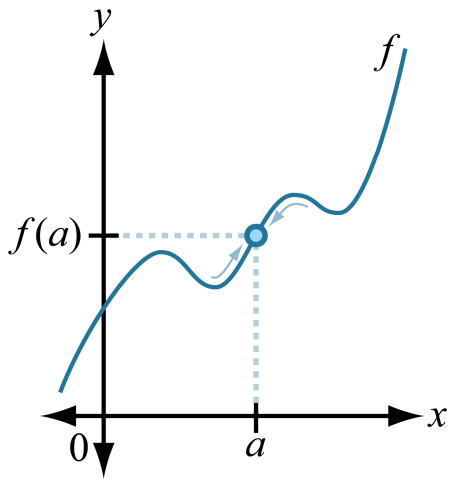
Continuity

Informal definition



Formal definition



Saying that f is continuous at $x = a$ means that

1. $f(a)$ exists
2. $\lim_{x \to a} f(x)$ exists
3. $f(a)$ and $\lim_{x \to a} f(x)$ equal each other.

Saying that f is continuous on an interval means that f is continuous at every x in the interval.

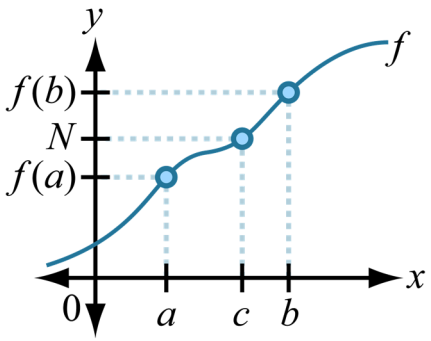
Properties

Suppose f and g are continuous, then the following are also continuous:

$$kf \qquad f + g \qquad f \cdot g \qquad \frac{f}{g}, \quad g(x) \neq 0$$

$$f \circ g \text{ and } g \circ f \text{, assuming no domain errors}$$

Intermediate value theorem

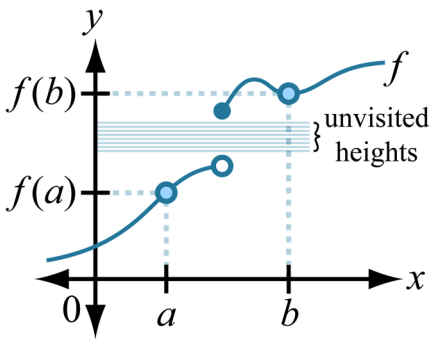


Hypotheses

1. f is continuous on $[a, b]$
2. $f(a) \neq f(b)$
3. $f(a) < N < f(b)$ or $f(b) < N < f(a)$

Conclusion

There is an $x = c$ on (a, b) where $f(c) = N$.



If f is discontinuous, the conclusion might or might not be satisfied. There is no guarantee either way.