Extrema

A maximum is a "greatest" value, and a minimum is a "least" value. The words "maximum" and "minimum" refer to y-values, not to the values of x at which they occur.

We say that f has a **local maximum** at $x = a$ when $f(a) \geq f(x)$ for every $x \neq a$ in a ball of finite radius centered at $x = a$.

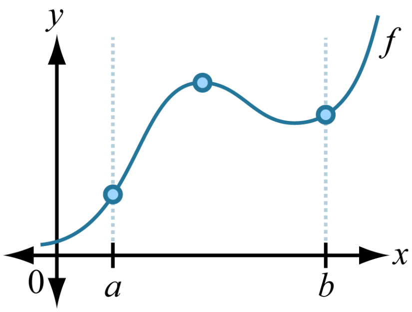
We say that f has a **local minimum** at $x = a$ when $f(a) \leq f(x)$ for every $x \neq a$ in a ball of finite radius centered at $x = a$.

We say that f has a **strict local maximum** at $x = a$ when $f(a) > f(x)$ for every $x \neq a$ in a ball of finite radius centered at $x = a$.

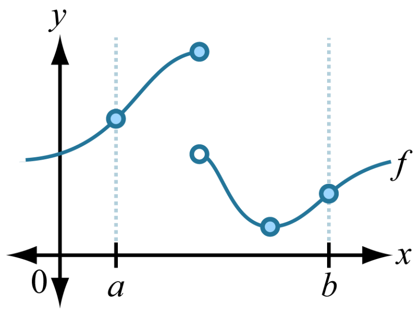
We say that f has a **strict local minimum** at $x = a$ when $f(a) < f(x)$ for every $x \neq a$ in a ball of finite radius centered at $x = a$.

We say that f has a **global (absolute) maximum** on a specified set of values of x at $x = a$ when $f(a) \geq f(x)$ for every $x \neq a$ in the specified set of values of x.

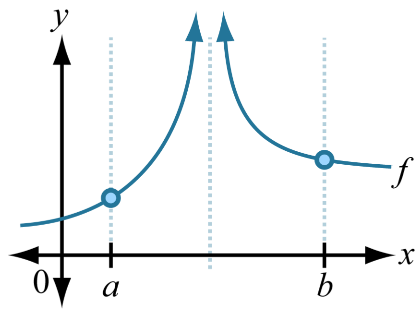
We say that f has a **global (absolute) minimum** on a specified set of values of x at $x = a$ when $f(a) \leq f(x)$ for every $x \neq a$ in that specified set of values of x.



Continuous
Has max and min



Discontinuous
Has max and min



Discontinuous
Does not have max

Extreme value theorem – If f is continuous on the interval $I = [a, b]$, then f yields a greatest value $f(c)$ and a least value $f(d)$ for some c and $d \in I$.

Finding global (absolute) extrema of a continuous function f on a set that has no open boundaries:

1. **State the domain** of interest.
2. Identify **closed endpoints** in the domain.
3. Identify **critical numbers** of f in the domain:
    a. $f'(x)$ DNE
    b. $f'(x) = 0$
4. Organizing results in a table, **evaluate** f at each critical number and at each endpoint.
5. **Circle** the **greatest** output value produced by f, wherever it occurs, and **circle** the **least** output value produced by f, wherever it occurs.
6. **Demonstrate use of calculus** by announcing that the global (absolute) extrema identified in step 5 were obtained "using the closed interval(s) method."