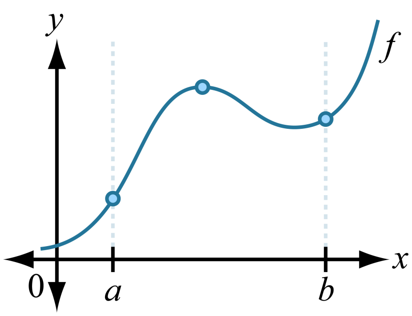
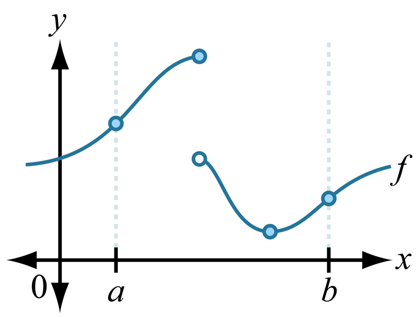
Extrema

A maximum is a "greatest" value, and a minimum is a "least" value. The words "maximum" and "minimum" refer to y-values, not to the values of x at which they occur.

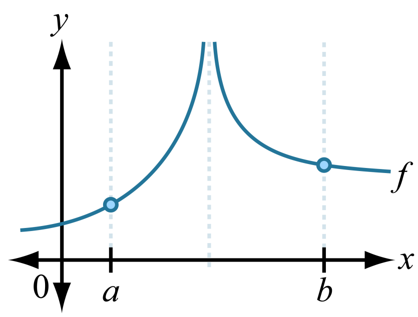
Vocabulary	Definition
f has a **local maximum** at $x = a$	$f(a) \geq f(x)$ for every x in an open interval containing $x = a$
f has a **local minimum** at $x = a$	$f(a) \leq f(x)$ for every x in an open interval containing $x = a$
f has a **strict local maximum** at $x = a$	$f(a) > f(x)$ for every $x \neq a$ in an open interval containing $x = a$
f has a **strict local minimum** at $x = a$	$f(a) < f(x)$ for every $x \neq a$ in an open interval containing $x = a$
f has a **global (absolute) maximum** at $x = a$	$f(a) \geq f(x)$ for every x in the domain of f
f has a **global (absolute) minimum** at $x = a$	$f(a) \leq f(x)$ for every x in the domain of f



Continuous
Has abs. max and abs. min



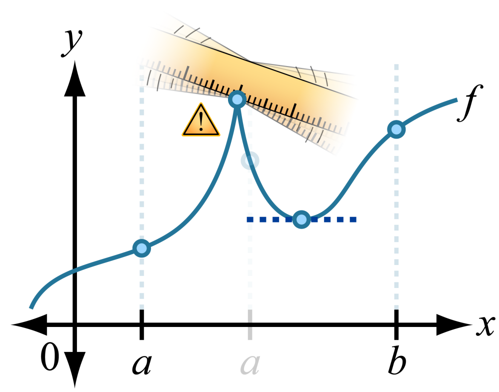
Discontinuous
Has abs. max and abs. min



Discontinuous
Does not have abs. max

Extreme value theorem	*Hypothesis*	*Conclusion*
	f is continuous on the interval $I = [a, b]$	1. f yields a greatest value $f(c)$ for some $c \in I$ 2. f yields a least value $f(d)$ for some $d \in I$

Candidates test for finding global (absolute) extrema of a continuous function f on a closed interval (or set of closed intervals)†



1. **State the domain** of interest.
2. **Indicate use of calculus** by writing, "Global (absolute) extrema can occur only at closed endpoints or at **critical points** (where, in the domain of f, $f'(x)$ DNE or $f'(x) = 0$) in the interior(s) of the closed interval(s)."
3. Identify **closed endpoints**.
4. Identify **critical numbers** of f in the interior(s) of the closed interval(s) of the domain.
5. Organizing results in a table, **evaluate** f at each critical number and at each endpoint.
6. **Circle** the **greatest** output value produced by f, wherever it occurs, and **circle** the **least** output value produced by f, wherever it occurs.

† When the set of interest is not a closed interval (or set of closed intervals), narrate analysis of a sign chart using language that demonstrates use of calculus principles (e.g. see variant of the first derivative test (handout 2.6 p. 2/Stewart 4th ed., p. 280)).