

Extrema

A maximum is a “greatest” value, and a minimum is a “least” value. The words “maximum” and “minimum” refer to y -values, not to the values of x at which they occur.

We say that f has a **local maximum** at $x = a$ when $f(a) \geq f(x)$ for every $x \neq a$ in a ball of finite radius centered at $x = a$.

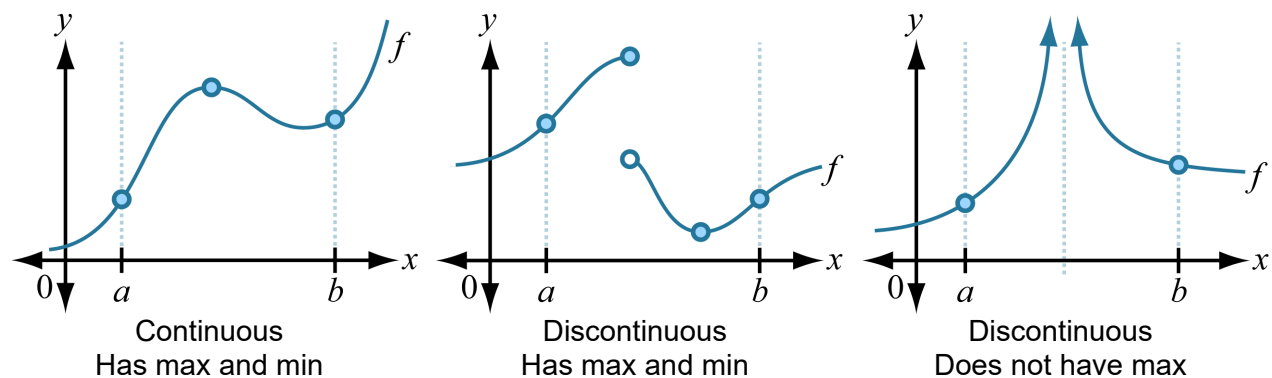
We say that f has a **local minimum** at $x = a$ when $f(a) \leq f(x)$ for every $x \neq a$ in a ball of finite radius centered at $x = a$.

We say that f has a **strict local maximum** at $x = a$ when $f(a) > f(x)$ for every $x \neq a$ in a ball of finite radius centered at $x = a$.

We say that f has a **strict local minimum** at $x = a$ when $f(a) < f(x)$ for every $x \neq a$ in a ball of finite radius centered at $x = a$.

We say that f has a **global (absolute) maximum** on a specified set of values of x at $x = a$ when $f(a) \geq f(x)$ for every $x \neq a$ in the specified set of values of x .

We say that f has a **global (absolute) minimum** on a specified set of values of x at $x = a$ when $f(a) \leq f(x)$ for every $x \neq a$ in that specified set of values of x .



Extreme value theorem – If f is continuous on the interval $I = [a, b]$, then f yields a greatest value $f(c)$ and a least value $f(d)$ for some c and $d \in I$.

Finding global (absolute) extrema of a continuous function f on a set that has no open boundaries:

1. **State the domain** of interest.
2. **Demonstrate use of calculus** by announcing, “Global (absolute) extrema can occur only at critical points (where $f'(x)$ DNE or where $f'(x) = 0$) or at endpoints.”
3. Identify **closed endpoints** in the domain.
4. Identify **critical numbers** of f in the domain:
 - a. $f'(x)$ DNE
 - b. $f'(x) = 0$
5. Organizing results in a table, **evaluate** f at each critical number and at each endpoint.
6. **Circle the greatest** output value produced by f , wherever it occurs, and **circle the least** output value produced by f , wherever it occurs.

