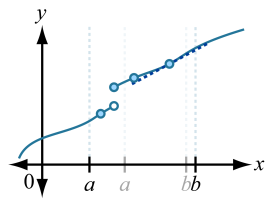
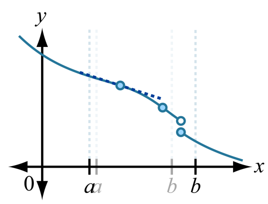
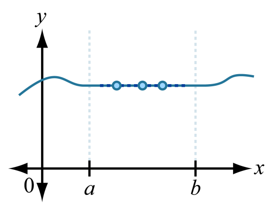
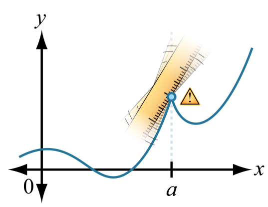
Use derivatives to describe features of graphs of functions

Use **first derivative** to describe local **slope**

In the following, the named situations occur when the corresponding defining conditions hold for every pair of x values x_{L} and x_{R} satisfying

- x_{L} and x_{R} are both on the interval of interest
- $x_{\mathrm{L}} < x_{\mathrm{R}}$

Name	**Definition**	**Graph of f**	Example **sufficient condition**
$f(x)$ is **incr**easing as x is increasing	$f(x_{\mathrm{L}}) < f(x_{\mathrm{R}})$		$f'(x) > 0$ on interval of interest
$f(x)$ is **decr**easing as x is increasing	$f(x_{\mathrm{L}}) > f(x_{\mathrm{R}})$		$f'(x) < 0$ on interval of interest
$f(x)$ is constant as x is increasing	$f(x_{\mathrm{L}}) = f(x_{\mathrm{R}})$		$f'(x) = 0$ on interval of interest (also necessary)
The graph of f has a cusp	Function is continuous, but left- and right-instantaneous slopes disagree at a point (so $f'(x)$ is undefined there)		

Justify answers on AP Calculus exams by appealing to calculus:

"f is increasing for all x on _____ (intervals) because $f'(x) > 0$ for all x on _____ (intervals)."

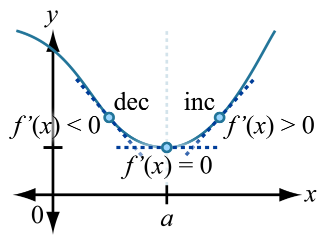
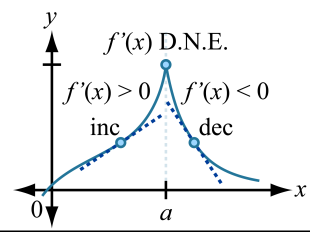
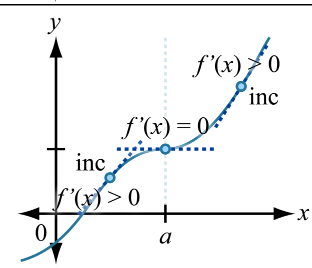
"f is decreasing for all x on _____ (intervals) because $f'(x) < 0$ for all x on _____ (intervals)."

"f is constant for all x on _____ (intervals) because $f'(x) = 0$ for all x on _____ (intervals)."

Use derivatives to describe features of graphs of functions

Use **changes in the first derivative** to identify **local extrema** at critical points of continuous functions

(“**First-derivative test**”)

Hypothesis	Graph of f	Conclusion
$f'(x)$ changes from **neg**ative to **pos**itive as x increases through critical number $x = a$		Strict **local minimum**
$f'(x)$ changes from **pos**itive to **neg**ative as x increases through critical number $x = a$		Strict **local maximum**
$f'(x)$ does not change sign as x increases through critical number $x = a$		**Neither** a **local minimum** nor a **local maximum**

Justify answers on AP Calculus exams by appealing to calculus:

“f has a relative maximum value of _____ (function value) at $x =$ ______ because $f'(x)$ changes from positive to negative at $x =$ ______.”

“f has a relative minimum value of _____ (function value) at $x =$ ______ because $f'(x)$ changes from negative to positive at $x =$ ______.”

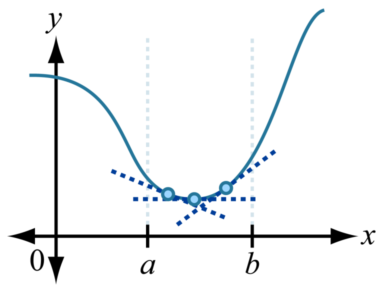
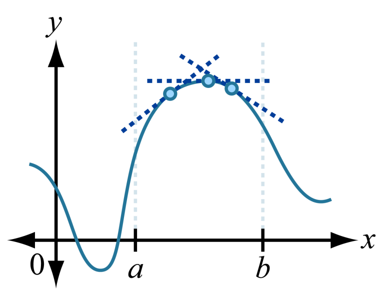
Variant of first derivative test useful for categorizing global (absolute) extrema on intervals that are missing a left endpoint, missing a right endpoint, or missing both endpoints

(see Stewart 4th ed., p. 280 for full statement)

“On _______ (interval), f has a global (absolute) maximum value of _____ (function value) at $x =$ ______ because $f'(x) > 0$ for all $x <$ ______ and $f'(x) < 0$ for all $x >$ ______.”

“On _______ (interval), f has a global (absolute) minimum value of _____ (function value) at $x =$ ______ because $f'(x) < 0$ for all $x <$ ______ and $f'(x) > 0$ for all $x >$ ______.”

Use derivatives to describe features of graphs of functions

Use **first derivative** and/or **second derivative** to describe **curvature**			
Name	**Definitions**	**Graph of f**	Examples of **sufficient conditions**
The graph of f is **concave up**	On interval of interest, graph of f lies above all of its tangents Stewart 4th ed. On open interval of interest, $f'(x)$ is increasing Abbreviated from Larson, Finney, and Hass textbooks		$f'(x)$ is increasing as x is increasing on interval of interest Main feature of definition in Larson, Finney, and Hass textbooks $f''(x) > 0$ on interval of interest
The graph of f is **concave down**	On interval of interest, graph of f lies below all of its tangents Stewart 4th ed. On open interval of interest, $f'(x)$ is decreasing Abbreviated from Larson, Finney, and Hass textbooks		$f'(x)$ is decreasing as x is increasing on interval of interest Main feature of definition in Larson, Finney, and Hass textbooks $f''(x) < 0$ on interval of interest

Justify answers on AP Calculus exams by appealing to calculus:

"f is ccu for all x on _______ (interval) because $f'(x)$ is increasing for all x on _______ (interval)."

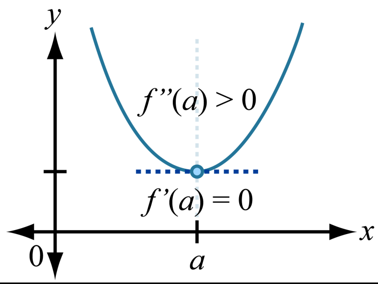
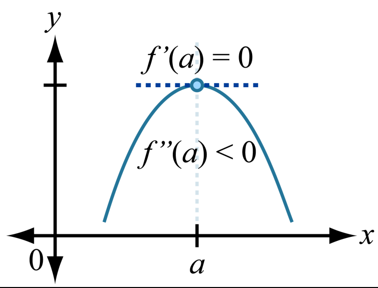
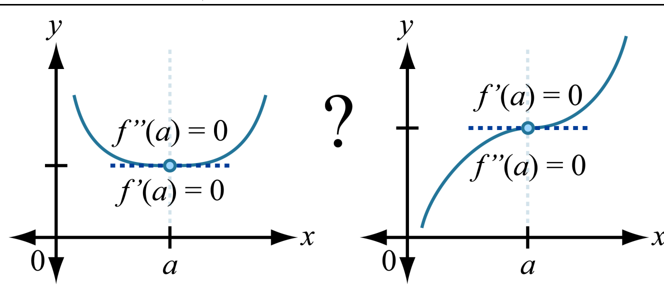
"f is ccd for all x on _______ (interval) because $f'(x)$ is decreasing for all x on _______ (interval)."

"f is ccu for all x on _______ (interval) because $f''(x) > 0$ for all x on _______ (interval)."

"f is ccd for all x on _______ (interval) because $f''(x) < 0$ for all x on _______ (interval)."

Use derivatives to describe features of graphs of functions

Use **second derivative** to identify **local extrema** of continuous functions
("**Second-derivative test**")

Hypothesis	Graph of f	Conclusion
1. $f'(a) = 0$ 2. $f''(a) > 0$		Strict **local minimum**
1. $f'(a) = 0$ 2. $f''(a) < 0$		Strict **local maximum**
1. $f'(a) = 0$ 2. $f''(a) = 0$		Inconclusive

Justify answers on AP Calculus exams by appealing to calculus:

"f has a relative minimum value of ____ (function value) at $x = a$ because $f'(a) = 0$ and $f''(a) > 0$."

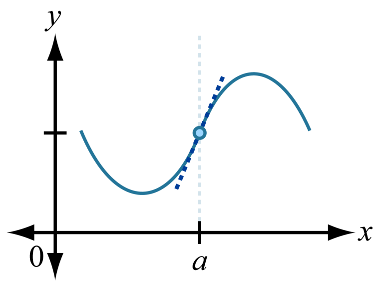
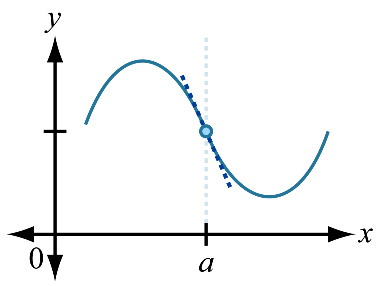
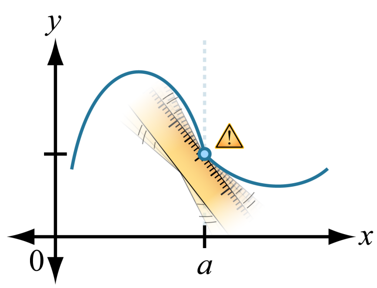
"f has a relative maximum value of ____ (function value) at $x = a$ because $f'(a) = 0$ and $f''(a) < 0$."

Use derivatives to describe features of graphs of functions

Use **changes in second derivative** to identify **points of inflection**

Thomas: **point of inflection** – point on a graph at which the graph changes from concave upward to concave downward or from concave downward to concave upward and at which a line tangent to the graph exists

Stewart, 4th ed., p. 244, and AP: **point of inflection** – point on a graph at which the graph changes from concave upward to concave downward or from concave downward to concave upward (implied: without a discontinuity)

Hypothesis	Graph of f	Name
$f''(x)$ changes from **pos**itive to **neg**ative as x increases through $x = a$ where tangent line exists		point of inflection
$f''(x)$ changes from **neg**ative to **pos**itive as x increases through $x = a$ where tangent line exists		
$f''(x)$ changes sign as x increases through $x = a$ where a tangent line does not exist		Thomas: no point of inflection Stewart and AP: point of inflection

Justify answers on exams in courses based on the Thomas calculus by appealing to calculus:

"f has a p.o.i. at (,) because $f''(x)$ changes from positive to negative at $x =$ ____ and a tangent line to the graph of f exists at (,)."

"f has a p.o.i. at (,) because $f''(x)$ changes from negative to positive at $x =$ ____ and a tangent line to the graph of f exists at (,)."

Justify answers on AP Calculus exams by appealing to calculus:

"f has a p.o.i. at (,) because $f''(x)$ changes from positive to negative at $x =$ ____."

"f has a p.o.i. at (,) because $f''(x)$ changes from negative to positive at $x =$ ____."