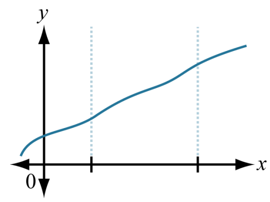
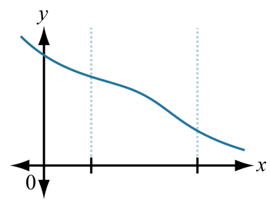
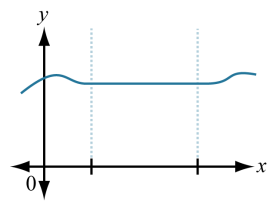
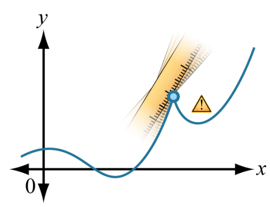
Use derivatives to describe features of graphs of functions

Use **first derivative** to describe local **slope**

In the following, the named situations occur when the corresponding defining conditions hold for every pair of x values a and b satisfying

- a and b are both on the interval of interest
- $a < b$

Name	Definition	Illustration	Example **sufficient condition**
$f(x)$ is strictly **incr**easing as x is increasing	$f(a) < f(b)$		$f'(x) > 0$ on interval of interest
$f(x)$ is strictly **decr**easing as x is increasing	$f(a) > f(b)$		$f'(x) < 0$ on interval of interest
$f(x)$ is constant as x is increasing	$f(a) = f(b)$		$f'(x) = 0$ on interval of interest (also necessary)
The graph of f has a cusp	Function is continuous, but left- and right-slopes disagree at a point (so $f'(x)$ is undefined there)		

Justify answers on AP Calculus exams by appealing to calculus:

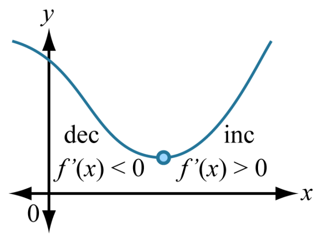
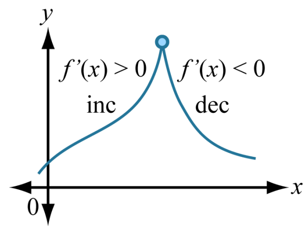
"f is increasing for all x on ______ (intervals) because $f'(x) > 0$ for all x on _____ (intervals)."

"f is decreasing for all x on ______ (intervals) because $f'(x) < 0$ for all x on _____ (intervals)."

"f is constant for all x on ______ (intervals) because $f'(x) = 0$ for all x on _____ (intervals)."

Use derivatives to describe features of graphs of functions

Use **changes in the first derivative** to identify **local extrema** of continuous functions ("**First-derivative test**")

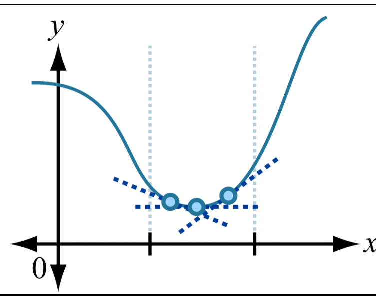
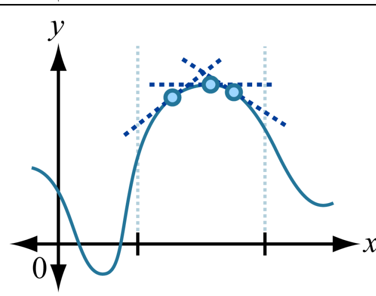
Hypothesis	Illustration	Conclusion
$f'(x)$ changes from **neg**ative to **pos**itive as x increases through a particular value		Strict **local minimum**
$f'(x)$ changes from **pos**itive to **neg**ative as x increases through a particular value		Strict **local maximum**

Justify answers on AP Calculus exams by appealing to calculus:

"f has a relative maximum value of ____ (function value) at $x =$ _____ because $f'(x)$ changes from positive to negative at $x =$ _____."

"f has a relative minimum value of ____ (function value) at $x =$ _____ because $f'(x)$ changes from negative to positive at $x =$ _____."

Use **second derivative** to describe **curvature**

Name	Definition	Illustration	Example **sufficient condition**
The graph of f is **conc**ave **up**	On the interval of interest, the graph of f lies above all of its tangents		$f''(x) > 0$ on interval of interest
The graph of f is **conc**ave **down**	On the interval of interest, the graph of f lies below all of its tangents		$f''(x) < 0$ on interval of interest

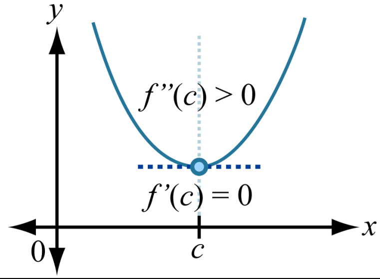
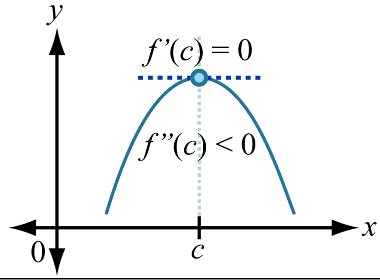
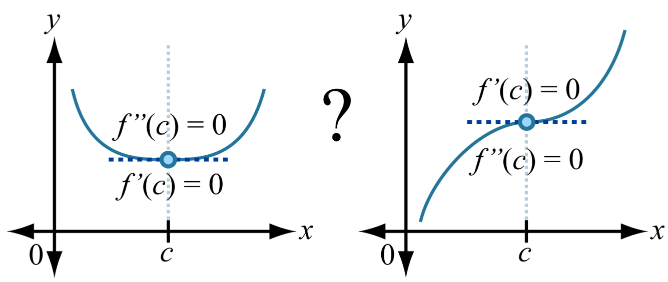
Justify answers on AP Calculus exams by appealing to calculus:

"f is ccu for all x on ______ (interval) because $f''(x) > 0$ for all x on _____ (interval)."

"f is ccd for all x on ______ (interval) because $f''(x) < 0$ for all x on _____ (interval)."

Use derivatives to describe features of graphs of functions

Use **second derivative** to identify **local extrema** of continuous functions
("**Second-derivative test**")

Hypotheses	Illustration	Conclusion
$f'(c) = 0$ and $f''(c) > 0$		Strict **local minimum**
$f'(c) = 0$ and $f''(c) < 0$		Strict **local maximum**
$f'(c) = 0$ and $f''(c) = 0$		Inconclusive

Justify answers on AP Calculus exams by appealing to calculus:

"f has a relative minimum value of ____ (function value) at $x = a$ because $f'(a) = 0$ and $f''(a) > 0$."

"f has a relative maximum value of ____ (function value) at $x = a$ because $f'(a) = 0$ and $f''(a) < 0$."

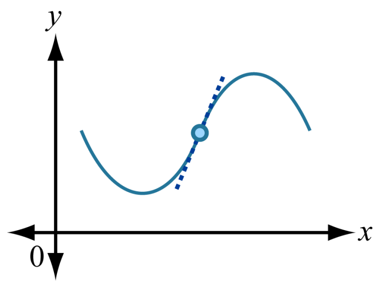
Use derivatives to describe features of graphs of functions

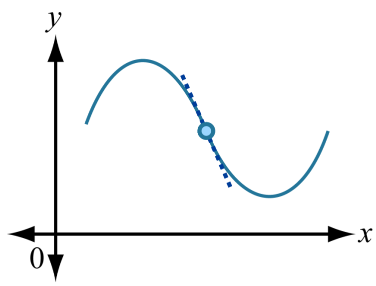
Use **changes in second derivative** to identify **points of inflection**

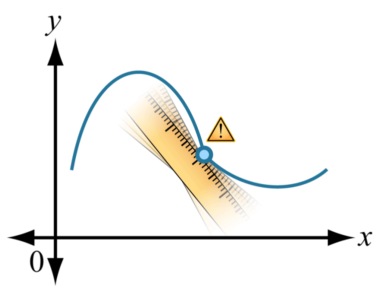
point of inflection – point on a graph at which the graph changes from concave upward to concave downward or from concave downward to concave upward and at which a line tangent to the graph exists*

* inclusion of existence of tangent line not universally specified (e.g. Stewart 4th ed. p. 244)

<table>
<tr><td>$f''(x)$ changes
from positive to negative as x increases through a particular value

at which
tangent line exists</td><td>
</td><td rowspan="2">point of inflection</td></tr>
<tr><td>$f''(x)$ changes
from negative to positive as x increases through a particular value

at which
tangent line exists</td><td>
</td></tr>
<tr><td>$f''(x)$ changes sign as x increases through a particular value

at which
a tangent line does not exist</td><td>
</td><td>no point of inflection</td></tr>
</table>

Justify answers on AP Calculus exams by appealing to calculus:

"f has a p.o.i. at (,) because $f''(x)$ changes from positive to negative at $x =$ ____ and a tangent line to the graph of f exists at (,)."

"f has a p.o.i. at (,) because $f''(x)$ changes from negative to positive at $x =$ ____ and a tangent line to the graph of f exists at (,)."