Curve sketching

1. Determine **domain** of f: (a) Determine x for which $f(x)$ is undefined. Do any values of x correspond to holes or vertical asymptotes?
   Find x-intercepts of $f(x)$: (b) Determine x for which $f(x) = 0$.
   Make a **sign chart** for f: ⟷ x
2. List any **known points** on the graph of f (e.g. obvious intercepts).
3. Study **end behavior** and identify any horizontal, slant, or polynomial asymptotes (consider how $f(x)$ behaves as $x \to \pm\infty$).
4. Find **critical numbers** of f: (a) Determine x for which is $f'(x)$ is undefined, and (b) Determine x for which $f'(x) = 0$.
   Make a **sign chart** for f': ⟷ x
5. Use the sign chart for f' to identify intervals on which $f(x)$ is **increasing or decreasing** with increasing x. At locations other than critical points of f, determine the sign of $f'(x)$.

$f'(x) > 0$	$f(x)$ is inc. with inc'ing x
$f'(x) < 0$	$f(x)$ is dec. with inc'ing x

   e.g. "f is increasing for all x on _____ (intervals) because $f'(x) > 0$ for all x on ____ (intervals)."
6. **Identify relative extrema**.
   e.g. "f has a relative (maximum/minimum) value of ____ (function value) at $x =$ _____ because $f'(x)$ changes from (positive/negative) to (negative/positive) at $x =$ _____."
7. Find **critical numbers** of f': (a) Determine x for which $f''(x)$ is undefined, and (b) Determine x for which $f''(x) = 0$.
   Make a **sign chart** for f'': ⟷ x
8. Use the sign chart for f'' to identify intervals on which the graph of f is **ccu or ccd**. At locations other than critical points of f', determine the sign of $f''(x)$.

$f''(x) > 0$	Graph of f is ccu
$f''(x) < 0$	Graph of f is ccd

   e.g. "f is ccu for all x on _____ (interval) because $f''(x) > 0$ for all x on _____ (interval)."
9. Use the sign chart for f'' to identify **points of inflection** for f.

If at x,	then,
• $f''(x)$ changes sign • tangent to graph of f exists	$(x, f(x))$ is p.o.i.

   e.g. "f has a p.o.i. at (,) because $f''(x)$ changes from ____tive to ____tive at $x =$ ____."
10. **Sketch** curve.
    a. Mark any **vertical asymptotes**.
    b. Draw any **known points**.
    c. Lightly sketch **end behavior**, if known.
    d. Lightly sketch a field of **sloped** line segments.
    e. Lightly sketch a field of **curved** segments locally consistent with the field of slopes.
    f. **Trace** a curve that
       i. Passes through the known points
       ii. Mimics every sketched arc segment through which it passes