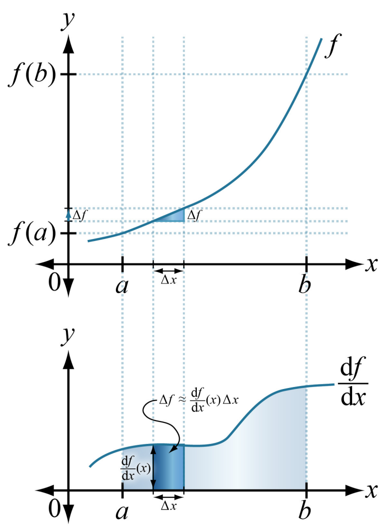
Fundamental theorem of calculus

Provided that the integrand is continuous on $[a, b]$, the **integral undoes** the **derivative**

$$\int_a^b \frac{\mathrm{d}f}{\mathrm{d}x}\,\mathrm{d}x = f(b) - f(a)$$

f is an "antiderivative" of $\frac{\mathrm{d}f}{\mathrm{d}x}$.

"The integral over an interval is equivalent to a function evaluated at the bounds."

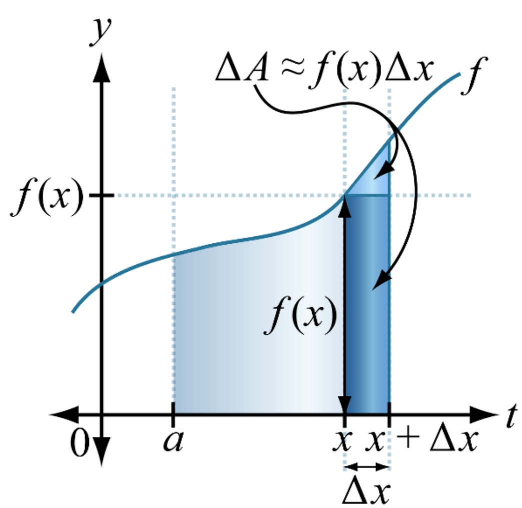


Provided that the integrand is continuous on $[a, b]$ and that $x \in [a, b]$, the **derivative undoes** the **integral**

$$\frac{\mathrm{d}}{\mathrm{d}x}\underbrace{\int_a^x f(t)\,\mathrm{d}t}_{A(x)} = f(x)$$

"The rate at which area is accumulated is the function."

$$\begin{aligned}\frac{\mathrm{d}A}{\mathrm{d}x} &= \lim_{\Delta x \to 0}\frac{A(x+\Delta x) - A(x)}{\Delta x}\\ &= \lim_{\Delta x \to 0}\frac{\Delta A}{\Delta x}\\ &= \lim_{\Delta x \to 0}\frac{f(x)\Delta x + \mathcal{O}(\Delta x^2)}{\Delta x}\\ &= f(x)\end{aligned}$$



Antiderivative

Because integrals undo derivatives, we can obtain integration formulas by reversing differentiation formulas.

Example: Power rule

$$\frac{\mathrm{d}}{\mathrm{d}x}[x^n + \text{any constant}] = nx^{n-1}$$

implies

$$\int_a^b x^n\,\mathrm{d}x = \left[\frac{x^{n+1}}{n+1} + C\right]_a^b, \qquad n \neq -1$$

When the $\int$ and $\mathrm{d}x$ appear without limits, they can together be read as "antiderivative" of whatever they surround. We abuse the $\int$ and $\mathrm{d}x$ symbols in the same way that we abuse symbols that denote derivatives. We apply the $\int$ and $\mathrm{d}x$ both (1) to names of functions and (2) to association rule formulas in terms of x.

Even though antiderivative identities can be derived from differentiation formulas, memorize a basic set of antidifferentiation identities (see separate table).

u-substitution

$$\int_{x=1}^{x=2} (x+1)^2\,\mathrm{d}x$$

$$\begin{aligned} u &= x + 1 & x &= 1\colon u = (1) + 1 = 2\\ \mathrm{d}u &= (1+0)\mathrm{d}x & x &= 2\colon u = (2) + 1 = 3\end{aligned}$$

$$\int_{u=2}^{u=3} u^2\,\mathrm{d}u$$