Fundamental theorem of calculus: Keeping track of functions

When encountering a problem type similar to one of the following, first make a list of statements that describe how a function of interest and its derivatives are related to a provided graph and its geometric features.

Problem type 1

Problem statement provides

The function of interest	What they give you instead
$g(x)$	Graph of function f $g(x) = \int_a^x f(t)\, \mathrm{d}t$

You write/think

Equations	Understanding
1. Copy the given statement. $g(x) = \int_a^x f(t)\, \mathrm{d}t$	The value of $g(x)$ is the signed area "under" the graph of f from $t = a$ to x
2. Differentiate both sides. $g'(x) = f(x)$	The derivative of g at x equals the height of the graph of f at $t = x$
3. Differentiate both sides. $g''(x) = f'(x)$	The second derivative (used to find curvature of graph of g) at x equals the slope of the graph of f at $t = x$

Problem type 2 (Superficial variation of problem type 1)

Problem statement provides

The function of interest	What they give you instead
$f(x)$	Graph of f' Value of $f(x_i)$

You write/think

Equations	Understanding
4. Integrate the equation in step 2. $f(x) - f(x_i) = \int_{x_i}^x \text{height of graph at } x\, \mathrm{d}x$	The change in the value of $f(x)$ corresponds to the accumulated signed area "under" the provided graph
2. Translate statement into equation. $f'(x) = \text{height of graph at } x$	**1. Say** The graph of f' is shown
3. Differentiate the equation in step 2. $f''(x) = \text{slope of graph at } x$	The concavity of f can be studied by studying the slope of the provided graph