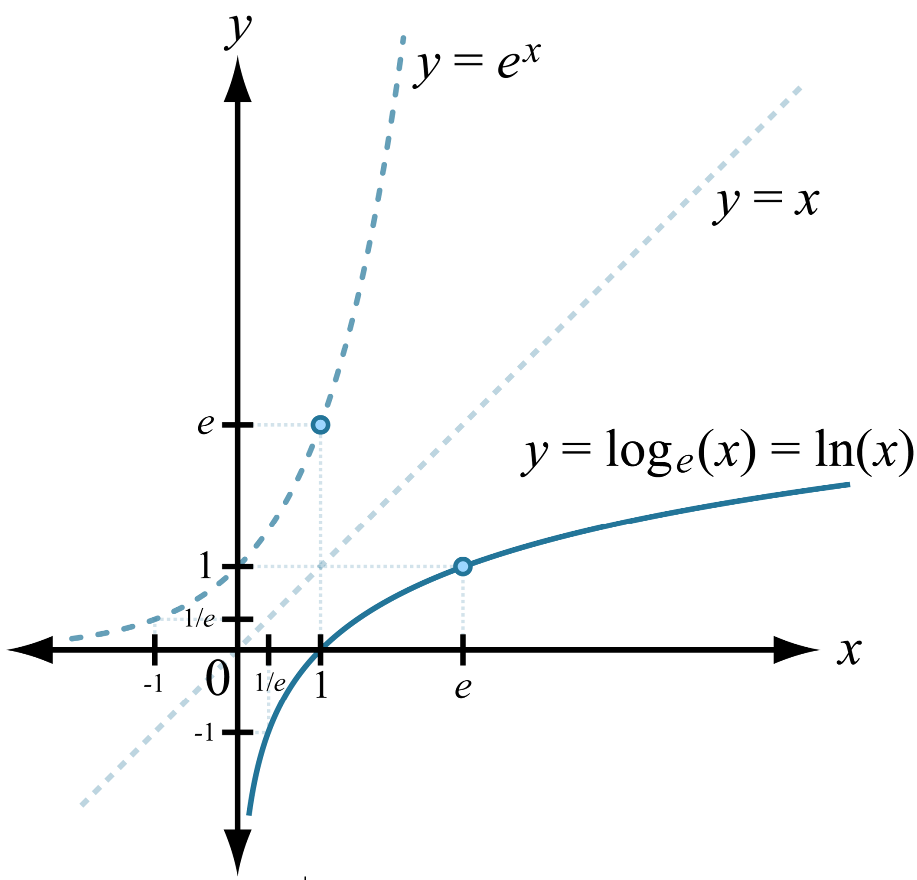
Calculus of exponentials and logs



$$\frac{\mathrm{d}}{\mathrm{d}x} b^x = \lim_{\Delta x \to 0} \frac{b^{x+\Delta x} - b^x}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{b^x b^{\Delta x} - b^x}{\Delta x}$$

$$\frac{\mathrm{d}}{\mathrm{d}x} b^x = b^x \underbrace{\left(\lim_{\Delta x \to 0} \frac{b^{\Delta x} - 1}{\Delta x} \right)}_{\substack{\text{if quantity} = 1, \\ \text{rename } b \text{ as } e}}$$

$$\frac{\mathrm{d}}{\mathrm{d}x} e^x = e^x$$

$$\int e^x \,\mathrm{d}x = e^x + C$$

$\ln(x)$ is the inverse of e^x

$$g'(x) = \frac{1}{f'\big(g(x)\big)}$$

$$\frac{\mathrm{d}}{\mathrm{d}x} \ln x = \frac{1}{e^{\ln x}} = \frac{1}{x}$$

$$\frac{\mathrm{d}}{\mathrm{d}x} \ln x = \frac{1}{x}$$

$$\frac{\mathrm{d}}{\mathrm{d}x} \ln|x| = \frac{\mathrm{d}}{\mathrm{d}x} \ln\sqrt{x^2}$$

$$= \frac{1}{e^{\ln\sqrt{x^2}}} \cdot \frac{1}{2}\left(x^2\right)^{-\frac{1}{2}} \cdot 2x$$

$$\int \frac{1}{x} \,\mathrm{d}x = \ln|x| + C$$

Change-of-base identities useful for calculating derivatives and integrals of exponential and logarithmic functions in terms of other bases:

$$b^x = \left(e^{\ln b}\right)^x \qquad \log_b x = \frac{\ln x}{\ln b}$$