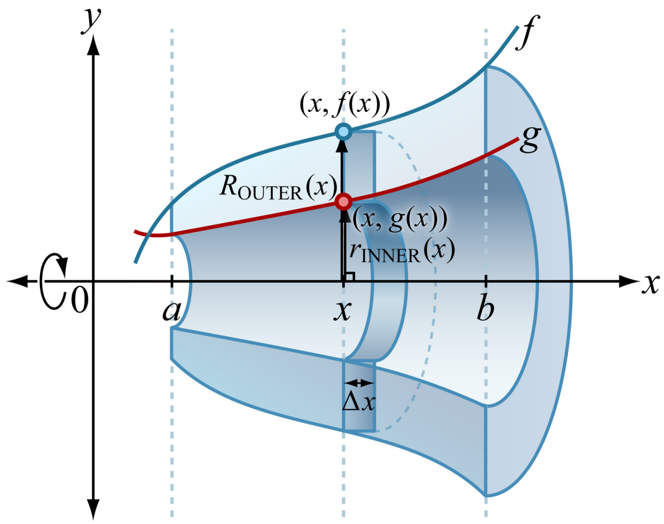
Using integrals to compute volumes

Volumes of solids of revolution

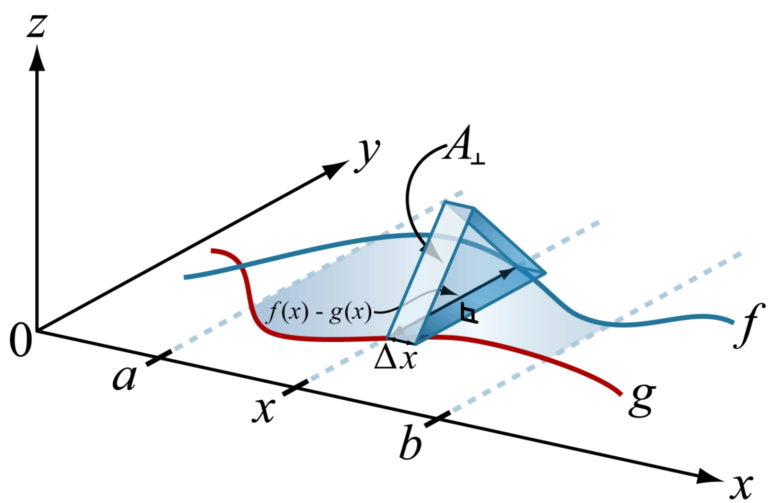


1. Draw a large figure. Large means that there is enough space to mark the figure up with comments.
2. Label axis of revolution (AOR).
3. If the axis of revolution is parallel to the y-axis, replace each x and y in the following discussion with y and x, respectively.
4. Draw a representative washer "at" x. Label its thickness Δx.
5. Label radius/radii. Draw all radii so that they extend perpendicularly outward *from* the AOR.
6. Label the outer endpoint of each radius arrow with a Cartesian ordered pair, substituting $y(x)$ in place of y.

$$\Delta V_{\text{WASHER}}(x) = \pi\left[R^2_{\text{OUTER}}(x) - r^2_{\text{INNER}}(x)\right]\Delta x$$

$$V = \pi \int_a^b \left[R^2_{\text{OUTER}}(x) - r^2_{\text{INNER}}(x)\right] \mathrm{d}x$$

Volumes of solids with specified cross sections



1. Draw a large figure. Large means that there is enough space to mark the figure up with comments.
2. Draw a representative slab "at" x. Label its thickness Δx.
3. Use Cartesian ordered pairs to label the points at the top left and bottom left corners (in the xy plane) of the slab.
4. Find an expression for the area $A(x)$ of the perpendicular face of the slab "at" x. It might be possible to express $A(x)$ in terms of $f(x) - g(x)$.

$$\Delta V_{\text{SLICE}}(x) = A_\perp(x)\Delta x$$

$$V = \int_a^b A_\perp(x)\,\mathrm{d}x$$