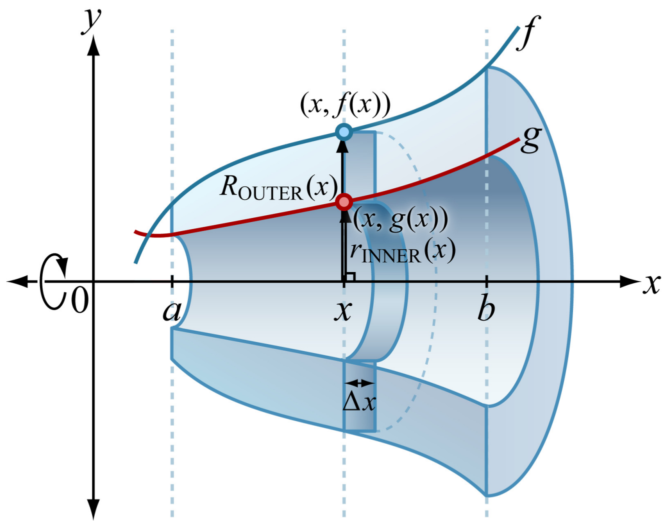
Using integrals to compute volumes

Volumes of solids of revolution

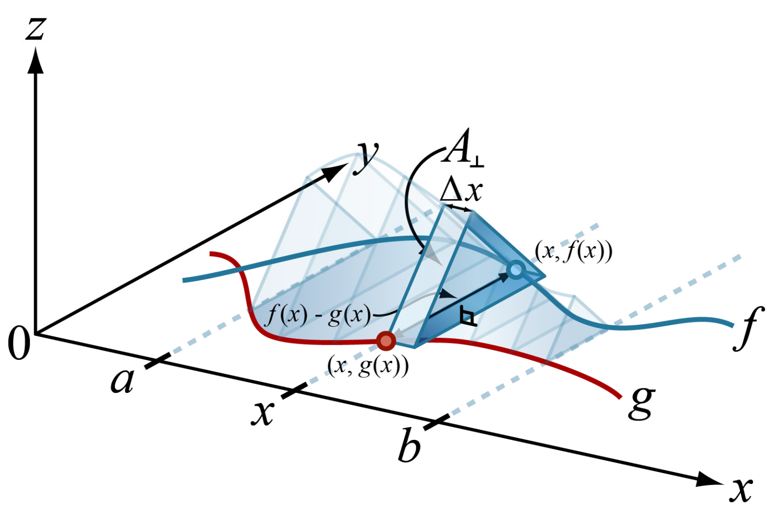


1. Draw a large figure. Large means that there is enough space to mark the figure up with comments.
2. Label axis of revolution (AOR).
3. If the axis of revolution is parallel to the y-axis, replace each x and y in the following discussion with y and x, respectively.
4. Draw a representative washer "at" x. Label its thickness Δx.
5. Label radius/radii. Draw all radii so that they extend perpendicularly outward *from* the AOR.
6. Label the outer endpoint of each radius arrow with a Cartesian ordered pair, substituting $y(x)$ in place of y.

$$\Delta V_{\text{WASHER}}(x) = \pi\left[R^2_{\text{OUTER}}(x) - r^2_{\text{INNER}}(x)\right]\Delta x$$

$$V = \pi \int_a^b \left[R^2_{\text{OUTER}}(x) - r^2_{\text{INNER}}(x)\right] \mathrm{d}x$$

Volumes of solids with specified cross sections



1. Draw a large figure. Large means that there is enough space to mark the figure up with comments.
2. Draw a representative slice of bread "at" x. Label its thickness Δx.
3. Use Cartesian ordered pairs to label the points at the top left and bottom left corners (in the xy plane) of the slice of bread.
4. Find an expression for the area $A_\perp(x)$ of the face (perpendicular to the x-axis) of the slice of bread "at" x. It might be possible to express $A_\perp(x)$ in terms of $f(x) - g(x)$.

$$\Delta V_{\text{SLICE}}(x) = A_\perp(x)\Delta x$$

$$V = \int_a^b A_\perp(x)\,\mathrm{d}x$$