Partial fractions and introduction to logistic growth

1. Solve

$$\int \frac{2x+3}{x^2+3x+2}\,\mathrm{d}x$$

Use u-substitution:

$$u = x^2+3x+2$$
$$\mathrm{d}u = (2x+3)\mathrm{d}x$$

$$\int \frac{2x+3}{x^2+3x+2}\,\mathrm{d}x = \int \frac{1}{u}\,\mathrm{d}u$$
$$= \ln|u| + C$$
$$= \ln|x^2+3x+2| + C$$

2. Solve

$$\int \frac{1}{x^2+3x+2}\,\mathrm{d}x$$

u-substitution will not work

$$\frac{1}{x^2+3x+2} = \frac{1}{(x+2)(x+1)}$$
$$= \frac{A}{x+2} + \frac{B}{x+1}$$
$$= \frac{A(x+1)}{(x+2)(x+1)} + \frac{B(x+2)}{(x+2)(x+1)}$$
$$\Rightarrow 1 = A(x+1) + B(x+2)$$

Let $x = -1$

$$1 = A\big((-1)+1\big) + B\big((-1)+2\big)$$
$$1 = A(0) + B(1)$$
$$1 = B$$

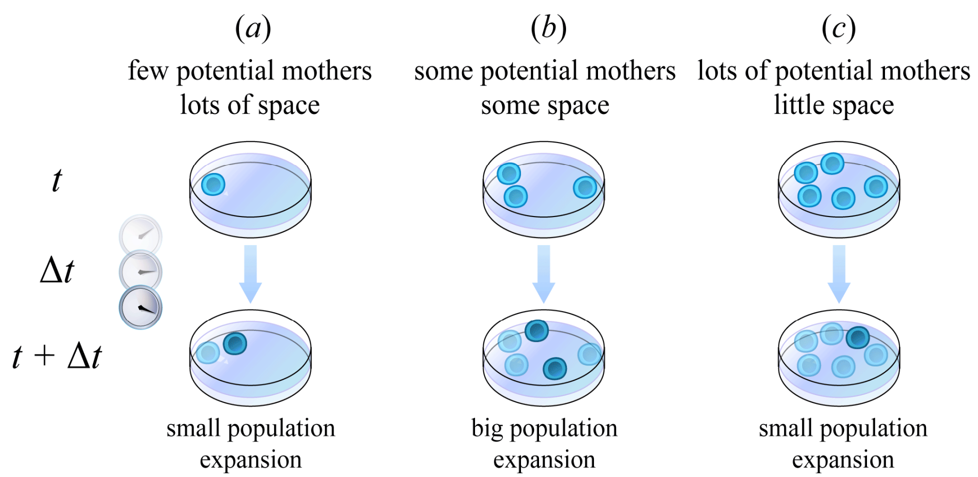
Let $x = -2$

$$1 = A\big((-2)+1\big) + B\big((-2)+2\big)$$
$$1 = A(-1) + B(0)$$
$$-1 = A$$

$$\frac{1}{x^2+3x+2} = \frac{-1}{x+2} + \frac{1}{x+1}$$
$$\int \frac{1}{x^2+3x+2}\,\mathrm{d}x = \int \left(\frac{-1}{x+2} + \frac{1}{x+1}\right)\mathrm{d}x$$
$$= \int \frac{-1}{x+2}\,\mathrm{d}x + \int \frac{1}{x+1}\,\mathrm{d}x$$
$$= -\ln|x+2| + \ln|x+1| + C$$
$$= \ln\frac{|x+1|}{|x+2|} + C$$

Ensure degree of numerator is less than degree of denominator before using this method (can use polynomial long division).

Logistic growth



P – **population** (number of individuals)

K – **carrying capacity** (number of individuals at which there is "no more space")

$$\Delta P \propto P(K-P)\Delta t$$
$$\frac{\mathrm{d}P}{\mathrm{d}t} \propto P(K-P)$$

Coefficients are usually written in the following way

$$\frac{\mathrm{d}P}{\mathrm{d}t} = rP\frac{(K-P)}{K}$$

Exercise: Solve for $P(t)$ given $P(0)$. Find $\lim_{t\to\infty} P(t)$ with and without using solution.