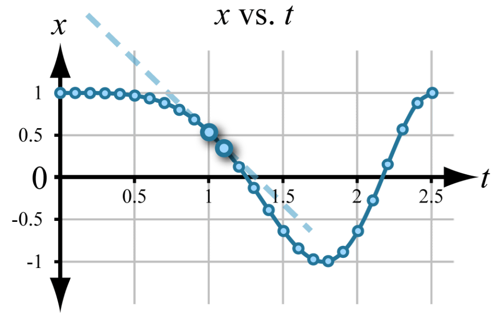
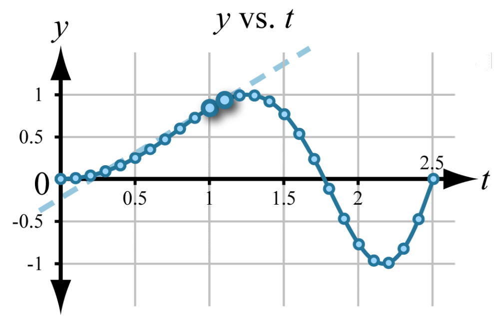
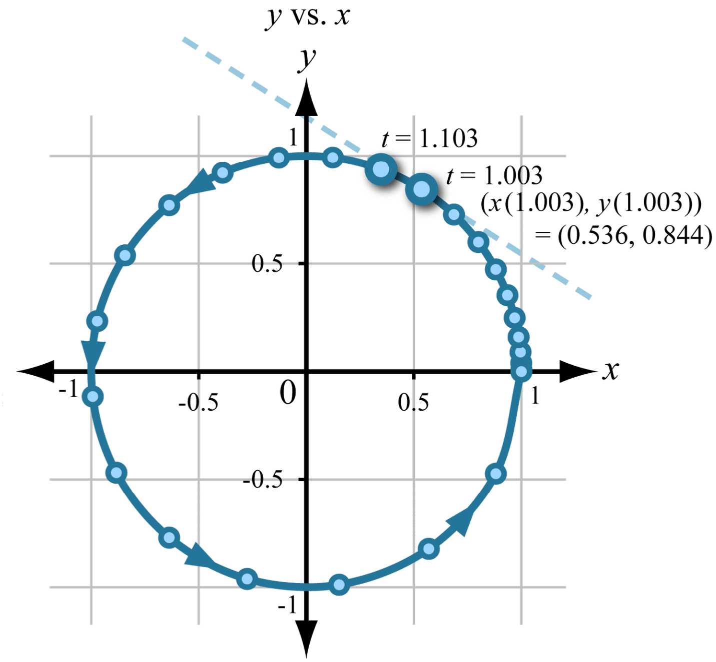
Parametrically-defined paths

When x and y are specified as functions of a parameter t, instead of in terms of each other, the relationship between x and y is said to be parametrically defined. Each value that the parameter t can take is like a house number, with the corresponding coordinate pair $(x(t), y(t))$ defining the actual geographic position of the house on a Cartesian grid. The relationship between x and y describes the set of points that defines the street.

t	$x = \cos(t^2)$	$y = \sin(t^2)$
0.000	1.000	0.000
0.100	1.000	0.010
0.201	0.999	0.040
0.301	0.996	0.090
⋮	⋮	⋮
0.702	0.881	0.473
0.802	0.800	0.600
0.902	0.686	0.727
1.003	0.536	0.844
1.103	0.347	0.938
1.203	0.123	0.992
1.303	-0.128	0.992
⋮	⋮	⋮







Static geometry

$$\frac{\mathrm{d}y}{\mathrm{d}t} = \frac{\mathrm{d}y}{\mathrm{d}x}\frac{\mathrm{d}x}{\mathrm{d}t}$$

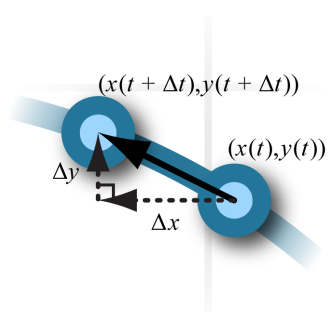
$$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{\frac{\mathrm{d}y}{\mathrm{d}t}}{\frac{\mathrm{d}x}{\mathrm{d}t}}, \quad \frac{\mathrm{d}x}{\mathrm{d}t} \neq 0$$

$$\frac{\mathrm{d}^2 y}{\mathrm{d}x^2} = \frac{\mathrm{d}\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)}{\mathrm{d}x}$$

$$\frac{\mathrm{d}^2 y}{\mathrm{d}x^2} = \frac{\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)}{\frac{\mathrm{d}x}{\mathrm{d}t}}, \quad \frac{\mathrm{d}x}{\mathrm{d}t} \neq 0$$

$$\int_{x=x_i}^{x=x_f} y(x)\,\mathrm{d}x = \int_{t=t_i}^{t=t_f} y(t)\,\frac{\mathrm{d}x}{\mathrm{d}t}\,\mathrm{d}t, \quad \text{also for } x \leftrightarrow y$$

Vector kinematics



Position	$\mathbf{r}(t) := \langle x(t), y(t)\rangle$
Velocity	$\mathbf{v}(t) := \langle x'(t), y'(t)\rangle$
Speed	$v(t) = \lVert\mathbf{v}(t)\rVert = \sqrt{[x'(t)]^2 + [y'(t)]^2}$
Acceleration	$\mathbf{a}(t) := \langle x''(t), y''(t)\rangle$