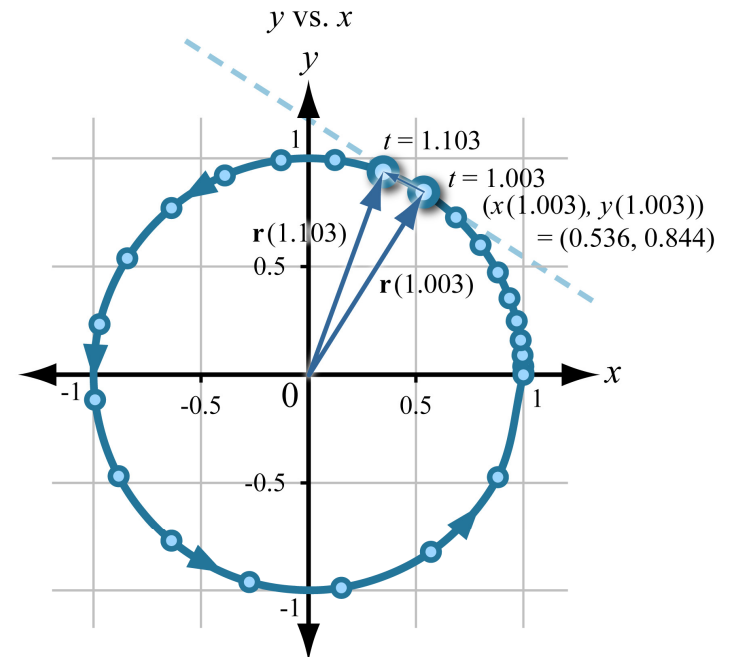
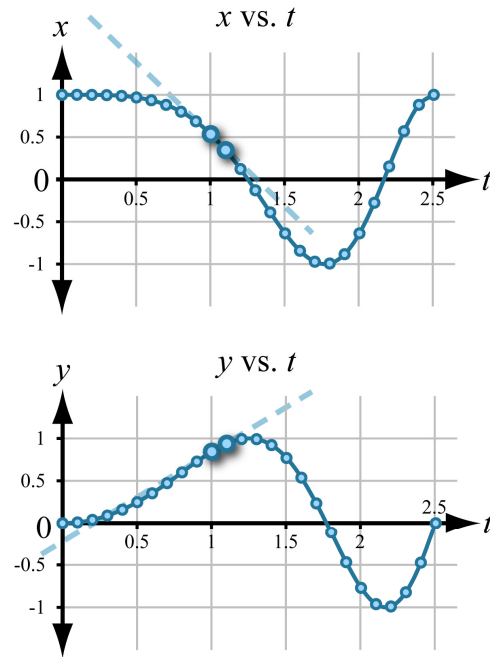


# Parametrically-defined paths

When  $x$  and  $y$  are specified as functions of a parameter  $t$ , instead of in terms of each other, the relationship between  $x$  and  $y$  is said to be parametrically defined. Each value that the parameter  $t$  can take is like a house number, with the corresponding coordinate pair  $(x(t), y(t))$  defining the actual geographic position of the house on a Cartesian grid. The relationship between  $x$  and  $y$  describes the set of points that defines the street.

$t$	$x = \cos(t^2)$	$y = \sin(t^2)$
0.000	1.000	0.000
0.100	1.000	0.010
0.201	0.999	0.040
0.301	0.996	0.090
⋮	⋮	⋮
0.702	0.881	0.473
0.802	0.800	0.600
0.902	0.686	0.727
1.003	0.536	0.844
1.103	0.347	0.938
1.203	0.123	0.992
1.303	-0.128	0.992
⋮	⋮	⋮



## Static geometry

$$\frac{dy}{dt} = \frac{dy}{dx} \frac{dx}{dt}$$

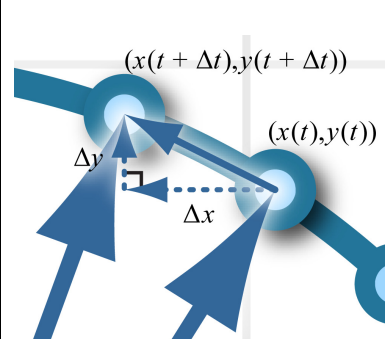
$$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}}, \quad \frac{dx}{dt} \neq 0$$

$$\frac{d^2y}{dx^2} = \frac{d\left(\frac{dy}{dx}\right)}{dx}$$

$$\frac{d^2y}{dx^2} = \frac{\frac{d}{dt}\left(\frac{dy}{dx}\right)}{\frac{dx}{dt}}, \quad \frac{dx}{dt} \neq 0$$

$$\int_{x=x_i}^{x=x_f} y(x) dx = \int_{t=t_i}^{t=t_f} y(t) \frac{dx}{dt} dt, \quad \text{also for } x \leftrightarrow y$$

## Vector kinematics



Position	$\vec{r}(t) := \langle x(t), y(t) \rangle$
Velocity	$\vec{v}(t) := \langle x'(t), y'(t) \rangle$
Speed	$v(t) = \ \vec{v}(t)\ $ $= \sqrt{[x'(t)]^2 + [y'(t)]^2}$
Acceleration	$\vec{a}(t) := \langle x''(t), y''(t) \rangle$