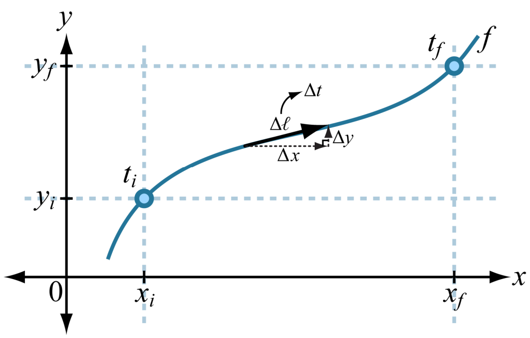
Arc length and area of surface of revolution

Arc lengths



$$\Delta\ell = \sqrt{(\Delta x)^2 + (\Delta y)^2}$$

Using $y(x)$

$$\Delta\ell = \sqrt{\left(\frac{\Delta x}{\Delta x}\right)^2 + \left(\frac{\Delta y}{\Delta x}\right)^2}\ \Delta x$$

$$\Delta\ell = \sqrt{1 + \left(\frac{\Delta y}{\Delta x}\right)^2}\ \Delta x$$

$$\ell = \int_{x=x_i}^{x=x_f} \sqrt{1 + \left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)^2}\ \mathrm{d}x$$

Using $x(y)$

$$\Delta\ell = \sqrt{\left(\frac{\Delta x}{\Delta y}\right)^2 + \left(\frac{\Delta y}{\Delta y}\right)^2}\ \Delta y$$

$$\Delta\ell = \sqrt{1 + \left(\frac{\Delta x}{\Delta y}\right)^2}\ \Delta y$$

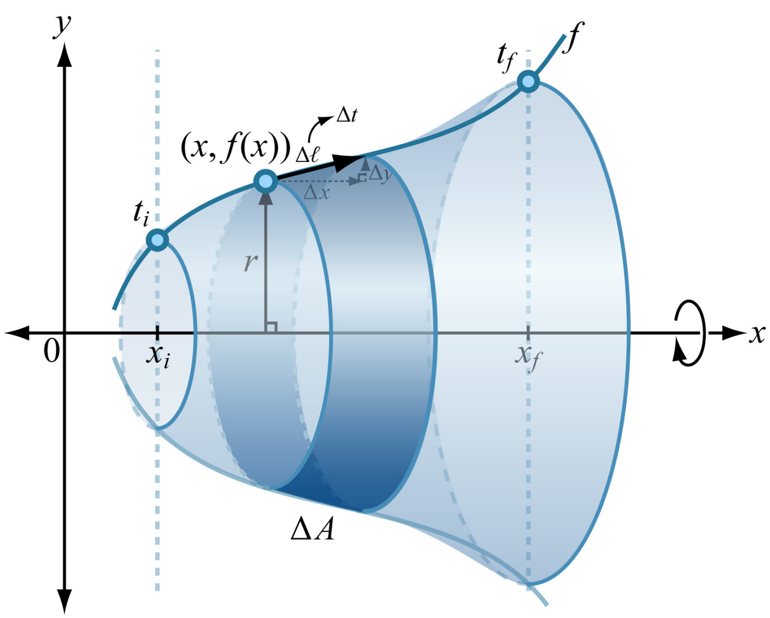
$$\ell = \int_{y=y_i}^{y=y_f} \sqrt{1 + \left(\frac{\mathrm{d}x}{\mathrm{d}y}\right)^2}\ \mathrm{d}y$$

Parametrically expressed

$$\Delta\ell = \sqrt{\left(\frac{\Delta x}{\Delta t}\right)^2 + \left(\frac{\Delta y}{\Delta t}\right)^2}\ \Delta t$$

$$\ell = \int_{t=t_i}^{t=t_f} \sqrt{\left(\frac{\mathrm{d}x}{\mathrm{d}t}\right)^2 + \left(\frac{\mathrm{d}y}{\mathrm{d}t}\right)^2}\ \mathrm{d}t$$

Areas of surfaces of revolution



Steps

1. Draw large figure with space for annotation.
2. Label axis of revolution (AOR).
3. Draw small strip.
4. Draw radius so that it extends ⊥ly outward *from* the AOR.
5. Label outer endpoint of radius with Cartesian ordered pair, substituting $y(x)$ in place of y.

$$\Delta A \approx (2\pi r)\Delta\ell$$

Revolved around x-axis

$$\Delta A \approx 2\pi y(x) \sqrt{1 + \left(\frac{\Delta y}{\Delta x}\right)^2}\ \Delta x$$

If the axis of revolution is parallel to the y-axis, replace each x in the following formula with y.

$$A = 2\pi \int_a^b r(x) \sqrt{1 + \left(\frac{\mathrm{d}r}{\mathrm{d}x}\right)^2}\ \mathrm{d}x$$

Parametrically expressed

$$\Delta A \approx 2\pi r(t) \sqrt{\left(\frac{\Delta x}{\Delta t}\right)^2 + \left(\frac{\Delta y}{\Delta t}\right)^2}\ \Delta t$$

$$A = 2\pi \int_{t=t_i}^{t=t_f} r(t) \sqrt{\left(\frac{\mathrm{d}x}{\mathrm{d}t}\right)^2 + \left(\frac{\mathrm{d}y}{\mathrm{d}t}\right)^2}\ \mathrm{d}t$$