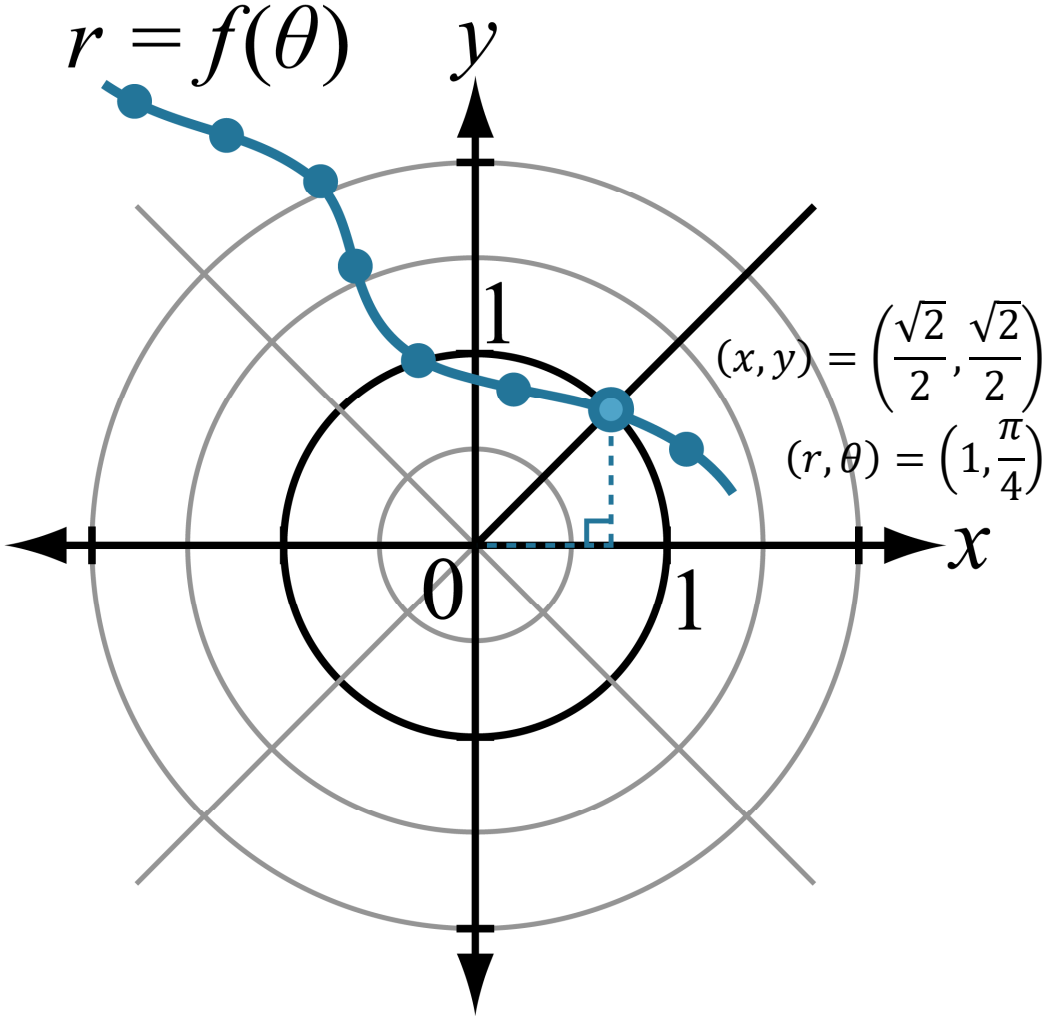
Calculus in polar coordinates

Points in 2-dimensional space can be identified using Cartesian and polar coordinates. Paths in 2-dimensional space can be described using functions expressed in terms of Cartesian coordinate variables and using functions expressed in terms of polar coordinate variables.



$$x = r\cos\theta$$

$$y = r\sin\theta$$

$$r = \pm\sqrt{x^2 + y^2}$$

$$\tan\theta = \frac{y}{x}$$

$$\frac{dy}{dx} = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}}$$

The standard inverse tangent function on many calculators provides only the principal angle. When obtaining polar coordinates for a point, use the signs of x and y to identify the quadrant containing the point. Use knowledge of the quadrant containing the point to determine whether the angle provided by the calculator needs to be corrected.

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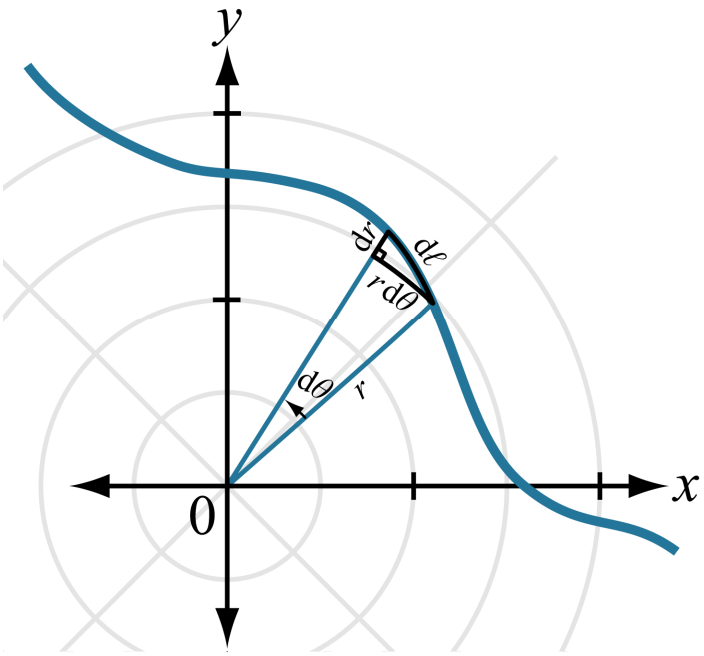
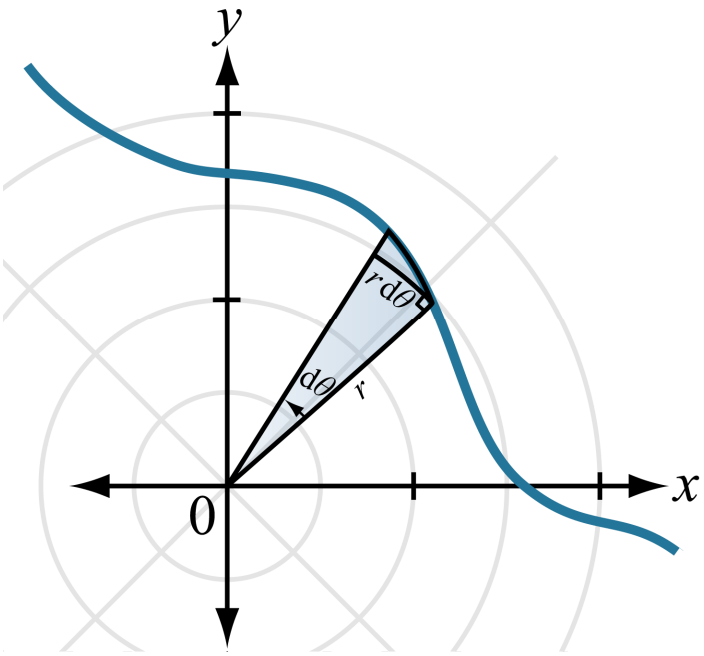
Arc length



$$(\mathrm{d}\ell)^2 = (\mathrm{d}r)^2 + (r\,\mathrm{d}\theta)^2$$

Area



$$\mathrm{d}A = \frac{1}{2}(r\,\mathrm{d}\theta)r = \frac{1}{2}r^2\,\mathrm{d}\theta$$