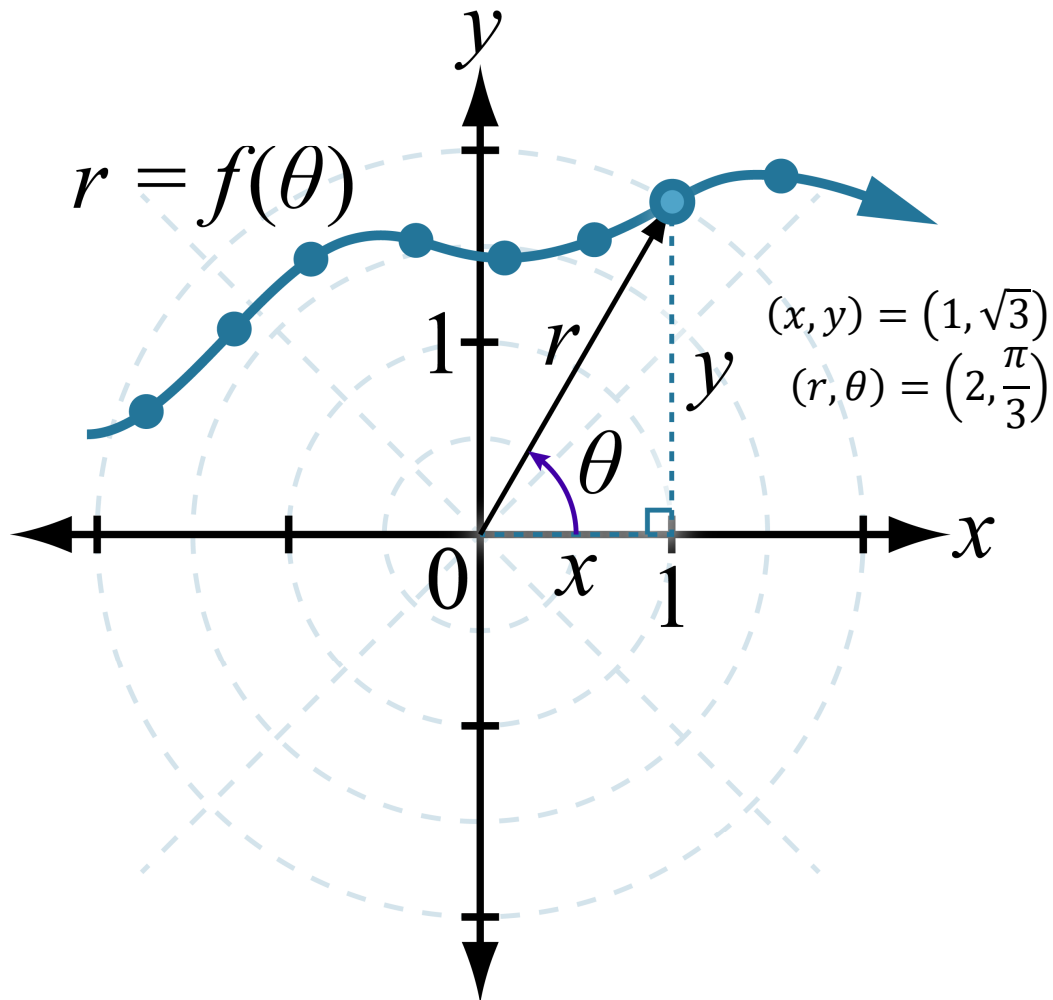


Calculus in polar coordinates

Points in 2-dimensional space can be identified using Cartesian and polar coordinates. Paths in 2-dimensional space can be described using functions expressed in terms of Cartesian coordinate variables and using functions expressed in terms of polar coordinate variables.



$$x = r \cos \theta$$

$$r = \pm\sqrt{x^2 + y^2}$$

$$y = r \sin \theta$$

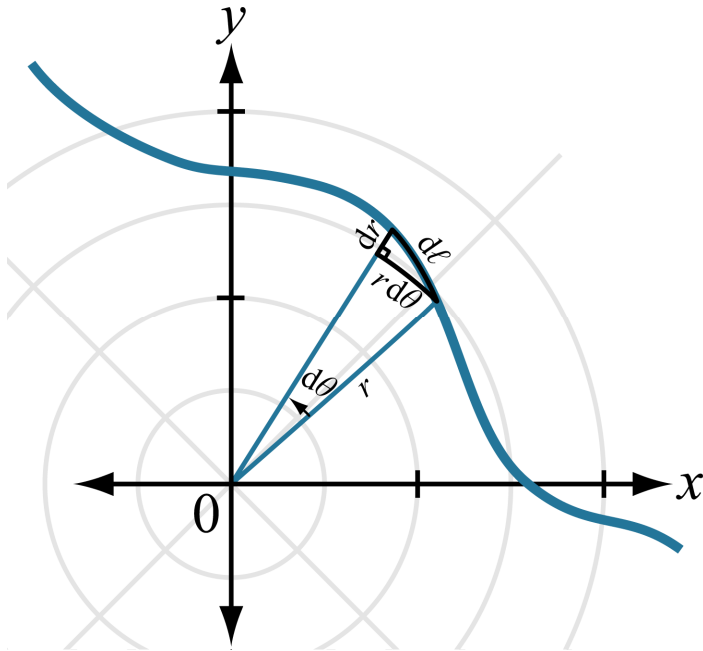
$$\tan \theta = \frac{y}{x}$$

$$\frac{dy}{dx} = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}}$$

The standard inverse tangent function on many calculators provides only the principal angle. When obtaining polar coordinates for a point, use the signs of x and y to identify the quadrant containing the point. Use knowledge of the quadrant containing the point to determine whether the angle provided by the calculator needs to be corrected.

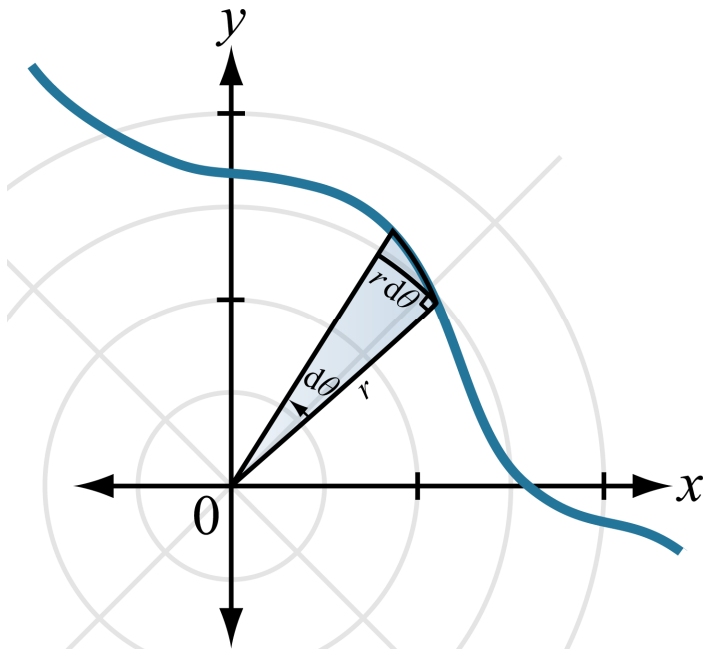
Calculus in polar coordinates

Arc length



$$(d\ell)^2 = (dr)^2 + (r d\theta)^2$$

Area



$$dA = \frac{1}{2}(r d\theta)r = \frac{1}{2}r^2 d\theta$$