Calculus tricks with Taylor series: Recognizing a summation

Sometimes you can recognize a particular summation as a power series for a function evaluated at a particular value of x.

$$f(x) = \sum_{n=0}^{\infty} \frac{f^{(n)}(a)}{n!}(x-a)^n$$

Example: Find the value of the series

$$\sum_{n=0}^{\infty} \frac{1}{n!}$$

We can recognize this summation as a power series representation of $f(x) = e^x$ evaluated at $x = 1$ in the following way.

$$\sum_{n=0}^{\infty} \frac{1}{n!} = \sum_{n=0}^{\infty} \frac{1}{n!} 1^n$$

$$= \left[\sum_{n=0}^{\infty} \frac{1}{n!} x^n \right]_{x=1}$$

$$= [e^x]_{x=1}$$

$$= e$$