

Problem solving SiQuENC (similar to Larkin, Knight, and Etkina)

Neatly and graphically represent situation(s)

1. Read the problem with careful attention.
2. Represent the problem statement in a visually navigable format.

Organize items like these	Using tools like these
a. Described situation(s)	a. Sketch(es)
b. Given information/quantities	b. Table(s)
c. Requested unknowns	c. List(s)

Make sure that your representation is complete so that you do not need to refer back to the original problem statement.

3. Use dashed bubble(s) to identify system(s) (set(s) of focal point(s) of analysis).
4. Label situations (e.g. initial and final, A and B, etc.).
5. Identify requested unknowns.

Graphically represent quantities and their relationships

Ways to represent quantities	Specific examples of types of diagrams
a. Lengths	a. Motion diagrams and graphs of kinematics quantities
b. Directions	b. Vector component decomposition diagrams
c. Countable copies of icons	c. Free body diagrams
	d. Bar charts
	e. Standing wave diagrams

Identify relevant allowed starting point (in) equation(s)

1. Look at cribsheets in which allowed knowledge has been organized into categories.
2. Try to identify (a) relevant categor(y)ies.
3. Look again at your representations of the problem statement and of quantities.
4. Write down relevant (in)equation(s).

Analyze

Deductively reason	Check whether result is reasonable
a. Manipulate algebraic equations.	a. “[X] has dimensions/units of _____, which is (not) expected.”
b. “According to the law of _____, [X] is (proportional to inversely proportional to etc.) [Y]. Because [X] _____, [Y] _____.”	b. Check order of magnitude.
c. Recognize an equation of a line $y = mx + b$ and interpret its slope and y -intercept.	c. Functional dependence: “According to this result, as [X] increases (by _____), [Y] _____. This is (not) reasonable because _____.”
d. Recognize what is being held constant and what is being changed.	d. Limiting cases: “According to this result, as [X] gets very _____, [Y] _____. This is (not) reasonable because _____.”
e. Design an experiment with correctly identified independent, dependent, and control variables.	

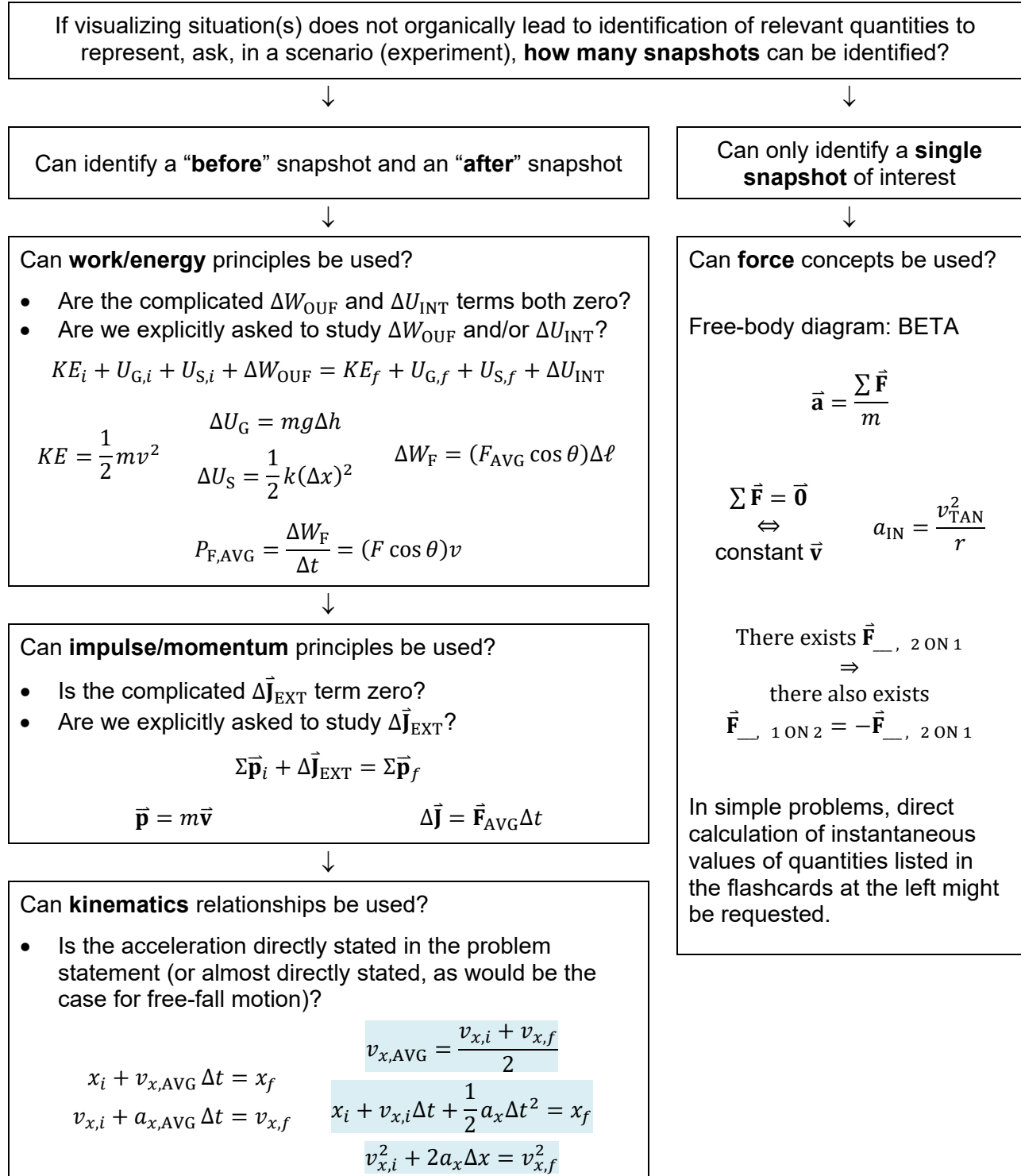
Communicate

1. Label each key logical point in your work with a number.
2. Following the order of your labels, translate each numbered key point into a sentence.

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Identify relevant quantities that permit efficient reasoning

The following flowchart can be used after kinematics, dynamics, momentum, and energy are studied. The flowchart is loosely based on ideas from the principles-of-mechanics flowchart on p. 137 of (Chi, et al., 1981) and the table of cues on p. 144 of the same reference.



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Analysis can be performed quantitatively and qualitatively

Features of qualitative reasoning are related to features of quantitative reasoning. Qualitative reasoning is distinct from quantitative reasoning, but qualitative reasoning can still be highly mathematical.

	Quantitative reasoning	Qualitative reasoning
Elements for representing snapshot features	Precise values	Zero Positive Negative
Comparisons	Precise differences (including a difference of zero)	Relatively large Relatively small Greater than Less than Equal to
Relationships	Algebraic association rules (formulas)	So-and-so's law states that y is proportional to x . Additional examples "y is inversely proportional to x " "Changing y does not change x " "y increases with x "
Operations for deducing solution sets	Arithmetic operations to simplify expressions and manipulate equations based on laws of algebra (e.g. addition property of equality, etc.)	According to so-and-so's law, y is proportional to x . Because (blah-blah-blah previous deduction), x is decreasing in this situation. This means that y is also decreasing.

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Presentation in the form of prose does not automatically confer upon reasoning the status of being qualitative.

	Both of these are examples of quantitative , not qualitative, reasoning	This is a mixture of qualitative and quantitative (mostly qualitative) reasoning
Traditional algebra	Natural-language narration of algebraic reasoning (looks like qualitative reasoning, but isn't)	Natural-language narration of mostly qualitative reasoning
$U_{G,i} = K_f$ $mgh = \frac{1}{2}mv^2$ $gh = \frac{1}{2}v^2$ $v = \sqrt{2gh}$ <p>Try $h = 1$</p> $v = \sqrt{2g}$ <p>Try $h = 2$</p> $v = \sqrt{2g \cdot 2} = \sqrt{4g}$ $\sqrt{4g} > \sqrt{2g}$ <p>So v increased</p>	<p>I set the initial gravitational potential energy equal to the final kinetic energy. The formula for gravitational potential energy is mgh, and the formula for kinetic energy is $\frac{1}{2}mv^2$. I divided the mass m out from both sides of the equation. I multiplied both sides of the equation by 2 and then took the positive root of both sides to solve for v. I substituted an example height of $h = 1$ and found that the resulting speed was $\sqrt{2g}$. When I substituted a greater example height of $h = 2$, I found that the resulting speed was now $\sqrt{4g}$, which is greater than $\sqrt{2g}$. This means that the v increased. Final velocity increases with increasing initial height.</p>	<p>Gravitational potential energy is mgh and, thus, proportional to h. This means that increasing the initial height from which the ball is dropped increases the initial gravitational potential energy. Air resistance is neglected, so the work performed on the ball-Earth system by external forces is zero. This means that mechanical energy is conserved so that all of the initial gravitational potential energy is converted to final kinetic energy at the bottom of the ball's fall. Thus, increasing the initial gravitational potential energy results in increasing the final kinetic energy. Because kinetic energy is $\frac{1}{2}mv^2$, increasing kinetic energy at a constant mass corresponds to increasing speed v. Thus, increasing the height from which the ball is dropped results in increasing the ball's speed at the bottom.</p>

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Remembering helpful sentences for communicating qualitative REASoNing

At the same time, Alice and Bob start running toward each other from opposite ends of a straight track. They meet on the track at a position closer to Bob's initial position than to Alice's initial position. Indicate who ran faster.

___ Alice ran faster ___ Bob ran faster ___ Alice and Bob ran equally fast

Briefly explain your reasoning.

REASoNing steps	Natural language (English)
State and walk through a relevant <u>r</u>elationship from allowed knowledge	According to the definition $v_{\text{AVG}} = \frac{\Delta \ell}{\Delta t}$, average speed is proportional to path length traveled and inversely proportional to elapsed travel time.
State what quantities, if any, are <u>e</u>qual, and why	Alice and Bob start running at the same time and meet at the same time, so Alice's elapsed travel time is the same as Bob's elapsed travel time.
State what quantities, if any, are <u>a</u>ltered or different, and why	The distance Alice travels is greater than the distance Bob travels because Alice and Bob meet at a point closer to Bob's initial position than to Alice's initial position.
So what?	Thus, Alice ran faster than Bob ran.
Is there any quantity to analyze <u>n</u>ext?	No quantities remain to be analyzed because we have already answered the question as requested.

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___ Alice ran faster ___ Bob ran faster ___ Alice and Bob ran equally fast

Briefly explain your reasoning.

Natural language (English)	Steps in deductive reasoning
According to the definition $v_{AVG} = \frac{\Delta \ell}{\Delta t}$, average speed is proportional to path length traveled and inversely proportional to elapsed travel time.	Postulate existence of variables v_{AVG} , $\Delta \ell$, and Δt for Alice and for Bob. Construct Table 1 by enumerating all* possible combinations of changes (or absences of changes) for these three variables. Use the relationship $v_{AVG} = \frac{\Delta \ell}{\Delta t}$ to reject possibilities 4, 5, 7, 8, 11, 12, 13, 15, 16, 17, 20, 21, 23, and 24. For example, reject possibility 4 because $\uparrow v_{AVG} = \frac{\text{same } \Delta \ell}{\uparrow \Delta t}$ doesn't make sense.
Alice and Bob start running at the same time and meet at the same time, so Alice's elapsed travel time is the same as Bob's elapsed travel time.	From the remaining possibilities, reject every possibility inconsistent with $\Delta t_A = \Delta t_B$. Reject possibilities in which $\Delta t_A < \Delta t_B$ (possibilities 1, 10, 19, 22, and 25), and reject possibilities in which $\Delta t_A > \Delta t_B$ (possibilities 3, 6, 9, 18, and 27).
The distance Alice travels is greater than the distance Bob travels because Alice and Bob meet at a point closer to Bob's initial position than to Alice's initial position.	From the remaining possibilities, reject every possibility inconsistent with $\Delta \ell_A > \Delta \ell_B$. Reject possibilities in which $\Delta \ell_A < \Delta \ell_B$ (possibility 2), and reject possibilities in which $\Delta \ell_A = \Delta \ell_B$ (possibility 14).
Thus, Alice ran faster than Bob ran.	(Translate the lone remaining possibility, possibility 26, into English).

* In AP Physics 1 and 2, students are usually not expected to consider *hormesis* or domain errors, so the outcome, if an outcome can be determined, of performing a binary comparison on a given variable will be "increase," "stay the same," or "decrease."

Table 1. Contingency chart

1.	$v_{AVG,A} < v_{AVG,B}$	$\Delta \ell_A < \Delta \ell_B$	$\Delta t_A < \Delta t_B$
2.	$v_{AVG,A} < v_{AVG,B}$	$\Delta \ell_A < \Delta \ell_B$	$\Delta t_A = \Delta t_B$
3.	$v_{AVG,A} < v_{AVG,B}$	$\Delta \ell_A < \Delta \ell_B$	$\Delta t_A > \Delta t_B$
4.	$v_{AVG,A} < v_{AVG,B}$	$\Delta \ell_A = \Delta \ell_B$	$\Delta t_A < \Delta t_B$
5.	$v_{AVG,A} < v_{AVG,B}$	$\Delta \ell_A = \Delta \ell_B$	$\Delta t_A = \Delta t_B$
6.	$v_{AVG,A} < v_{AVG,B}$	$\Delta \ell_A = \Delta \ell_B$	$\Delta t_A > \Delta t_B$
7.	$v_{AVG,A} < v_{AVG,B}$	$\Delta \ell_A > \Delta \ell_B$	$\Delta t_A < \Delta t_B$
8.	$v_{AVG,A} < v_{AVG,B}$	$\Delta \ell_A > \Delta \ell_B$	$\Delta t_A = \Delta t_B$
9.	$v_{AVG,A} < v_{AVG,B}$	$\Delta \ell_A > \Delta \ell_B$	$\Delta t_A > \Delta t_B$
10.	$v_{AVG,A} = v_{AVG,B}$	$\Delta \ell_A < \Delta \ell_B$	$\Delta t_A < \Delta t_B$
11.	$v_{AVG,A} = v_{AVG,B}$	$\Delta \ell_A < \Delta \ell_B$	$\Delta t_A = \Delta t_B$
12.	$v_{AVG,A} = v_{AVG,B}$	$\Delta \ell_A < \Delta \ell_B$	$\Delta t_A > \Delta t_B$
13.	$v_{AVG,A} = v_{AVG,B}$	$\Delta \ell_A = \Delta \ell_B$	$\Delta t_A < \Delta t_B$
14.	$v_{AVG,A} = v_{AVG,B}$	$\Delta \ell_A = \Delta \ell_B$	$\Delta t_A = \Delta t_B$
15.	$v_{AVG,A} = v_{AVG,B}$	$\Delta \ell_A = \Delta \ell_B$	$\Delta t_A > \Delta t_B$
16.	$v_{AVG,A} = v_{AVG,B}$	$\Delta \ell_A > \Delta \ell_B$	$\Delta t_A < \Delta t_B$
17.	$v_{AVG,A} = v_{AVG,B}$	$\Delta \ell_A > \Delta \ell_B$	$\Delta t_A = \Delta t_B$
18.	$v_{AVG,A} = v_{AVG,B}$	$\Delta \ell_A > \Delta \ell_B$	$\Delta t_A > \Delta t_B$
19.	$v_{AVG,A} > v_{AVG,B}$	$\Delta \ell_A < \Delta \ell_B$	$\Delta t_A < \Delta t_B$
20.	$v_{AVG,A} > v_{AVG,B}$	$\Delta \ell_A < \Delta \ell_B$	$\Delta t_A = \Delta t_B$
21.	$v_{AVG,A} > v_{AVG,B}$	$\Delta \ell_A < \Delta \ell_B$	$\Delta t_A > \Delta t_B$
22.	$v_{AVG,A} > v_{AVG,B}$	$\Delta \ell_A = \Delta \ell_B$	$\Delta t_A < \Delta t_B$
23.	$v_{AVG,A} > v_{AVG,B}$	$\Delta \ell_A = \Delta \ell_B$	$\Delta t_A = \Delta t_B$
24.	$v_{AVG,A} > v_{AVG,B}$	$\Delta \ell_A = \Delta \ell_B$	$\Delta t_A > \Delta t_B$
25.	$v_{AVG,A} > v_{AVG,B}$	$\Delta \ell_A > \Delta \ell_B$	$\Delta t_A < \Delta t_B$
26.	$v_{AVG,A} > v_{AVG,B}$	$\Delta \ell_A > \Delta \ell_B$	$\Delta t_A = \Delta t_B$
27.	$v_{AVG,A} > v_{AVG,B}$	$\Delta \ell_A > \Delta \ell_B$	$\Delta t_A > \Delta t_B$

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Textbook correlation

This step sheet is based on problem-solving processes similar to the one described by Larkin *et al.* in 1980.^{1,2} The chart below illustrates correlations between the SiQuENC steps and problem-solving steps from Larkin,^{1,2} Etkina³ (978-0-321-87972-1), and Knight⁴ (978-0-321-87972-1).

Larkin <i>et al.</i> ^{1,2}	Knight ⁴	Etkina ³	SiQuENC
“(i) If the problem statement is not accompanied by a picture, . . . sketch one.”	Prepare	Sketch and translate	Neatly and graphically represent <u>situation(s)</u>
“(ii) Selecting tentatively a set of principles to use, . . . construct an abstract problem representation containing physical entities (such as forces and energies) relevant to those principles.”		Simplify and diagram	Graphically represent <u>quantities</u> and their relationships
“(iii) . . . rerepresent the problem as a set of equations.”	Solve	Represent mathematically	Identify relevant allowed starting point <u>equations</u>
	Assess	Solve and evaluate	<u>Analyze</u>
			<u>Communicate</u>

This table illustrates only four examples of the many references^{1,2,5,6,7,8,9,10,11,12,13,14,15,16,3,17,4} that students and teachers can use to support the use of multiple representations and qualitative reasoning in problem-solving in physics.

Acknowledgments

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References

- ¹ J. Larkin, J. McDermott, D.P. Simon, and H.A. Simon, *Science* (80-.). **208**, 1335 (1980).
- ² M.T.H. Chi, P.J. Feltovich, and R. Glaser, *Cogn. Sci.* **5**, 121 (1981).
- ³ E. Etkina, M.J. Gentile, and A. Van Heuvelen, *College Physics* (Pearson, Glenview, IL, 2014).
- ⁴ R.D. Knight, B. Jones, and S. Field, *College Physics: A Strategic Approach*, 3rd ed. (Pearson, Glenview, IL, 2015).
- ⁵ F. Reif, *Phys. Teach.* **19**, 310 (1981).
- ⁶ A. Van Heuvelen, *Am. J. Phys.* **59**, 891 (1991).
- ⁷ A. Van Heuvelen, *Am. J. Phys.* **59**, 898 (1991).
- ⁸ F. Reif, *Am. J. Phys.* **63**, 17 (1995).
- ⁹ W.J. Leonard, R.J. Dufresne, and J.P. Mestre, *Am. J. Phys.* **64**, 1495 (1996).
- ¹⁰ R.J. Dufresne, W.J. Gerace, and W.J. Leonard, *Phys. Teach.* **35**, 270 (1997).

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- ¹¹ A. Van Heuvelen and X. Zou, Am. J. Phys. **69**, 184 (2001).
- ¹² K.A. Harper, Phys. Teach. **44**, 250 (2006).
- ¹³ R. Mualem and B.-S. Eylon, Phys. Teach. **45**, 158 (2007).
- ¹⁴ D. Rosengrant, A. Van Heuvelen, and E. Etkina, Phys. Rev. Spec. Top. - Phys. Educ. Res. **5**, 1 (2009).
- ¹⁵ D. Rosengrant, Phys. Teach. **49**, 36 (2011).
- ¹⁶ K.A. Harper, Phys. Teach. **50**, 424 (2012).
- ¹⁷ E. Etkina, Am. J. Phys. **83**, 669 (2015).