K8

Title UAM equations

Ingredients

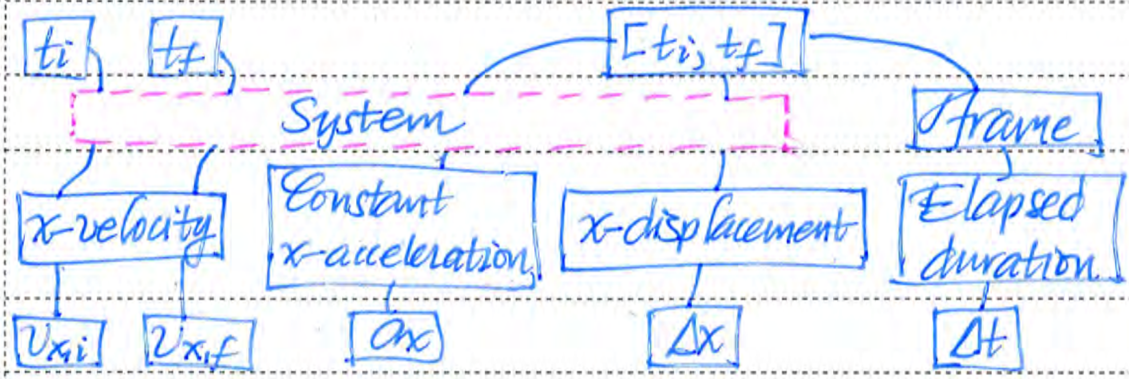
Sketch

At/Through

Owner

Quantity

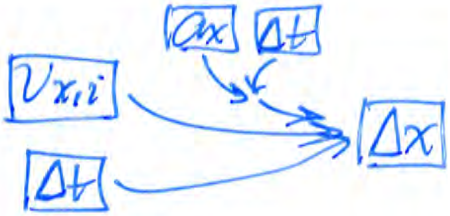
Variable

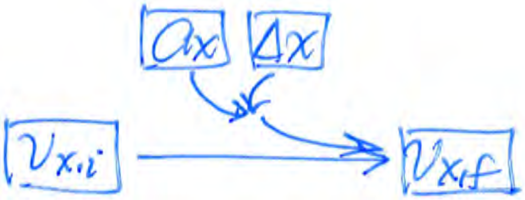


Giver

Recipe

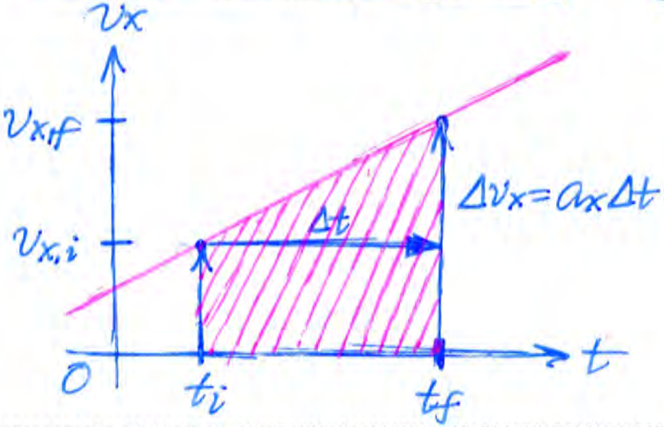
Diagram the relationship





Graphically present quantities

vx-t plot sometimes allows avoiding some algebra



Beginning-Middle-End Chart

$t_i =$

$x_i =$	$y_i =$
$v_{x,i} =$	$v_{y,i} =$

$[t_i; t_f]$

$a_x =$	$a_y =$

$t_f =$

$x_f =$	$y_f =$
$v_{x,f} =$	$v_{y,f} =$

Mathematical relationship

$$v_{x,i}\Delta t + \frac{1}{2}a_x\Delta t^2 = \Delta x \qquad v_{x,i}^2 + 2a_x\Delta x = v_{x,f}^2$$

Recipe number **K8**: The **title** of this recipe sheet is "**UAM equations**".

The top half of this sheet consists of an "**Ingredients**" section with a row labeled "Sketch", a row labeled "At/Through", a row labeled "Owner", a row labeled "Quantity", a row labeled "Variable", and a row labeled "Giver." In this sheet, the row labeled "Giver" isn't used.

For the "Sketch", draw three snapshots showing a cart moving toward the right across a firm surface. Draw trailing motion-blur streaks or so-called "whooshies" to emphasize instantaneous motion in each snapshot. Draw a dashed bubble around the earliest snapshot of the cart, at the left, to indicate that the cart is the so-called "System". Draw an arrow labeled +x to indicate that the positive-x direction points to the right.

In the rows of the "Ingredients" section other than the row for the sketch, document the following relationships, using flowchart paths, if helpful: There are two "Owners": One is the "System", and the other is the "Frame". At initial time t_i (t-sub-i), the "System" has the "Quantity" initial "x-velocity" denoted $v_{x,i}$ (v-sub-x-i), and at final time t_f (t-sub-f), the "System" has the "Quantity" final "x-velocity" $v_{x,f}$. Through the interval from initial time t_i (t-sub-i) to final time t_f (t-sub-f), the "System" has the "Quantity" "Constant x-acceleration" denoted a_x (a-sub-x). Through the same interval from initial time t_i (t-sub-i) to final time t_f (t-sub-f), the "System" also accrues the "Quantity" "x-displacement" denoted (Delta x). Also for the same interval from initial time t_i (t-sub-i) to final time t_f (t-sub-f), the "Frame", meaning the collection of rulers and clocks used to make measurements and referred together as the "frame of reference", has the "Quantity" "Elapsed duration" denoted by the "Variable" (Delta t).

The bottom half of this sheet consists of a "**Recipe**" section with a row labeled "Diagram the relationship", a row labeled "Graphically present quantities", and a row labeled "Mathematical relationship".

The row labeled "Diagram the relationship" will be divided into two sections.

In the first section, draw a flow chart showing that initial x-velocity $v_{x,i}$ (v-sub-x-i) contributes to accrued x-displacement (Delta x) and that elapsed duration (Delta t) also contributes to accrued x-displacement (Delta x). In this same flow chart, show that constant x-acceleration a_x (a-sub-x) and another copy of elapsed duration (Delta t) both contribute to a quantity that also contributes to accrued x-displacement (Delta x). Recite a story: "If the system were to maintain the initial x-velocity through some elapsed duration, this alone would be enough for the system to accrue some x-displacement, but beyond that, if the x-velocity were also in the midst of changing at constant x-acceleration through that same elapsed duration, the accrued x-displacement would be different from the amount that would be accrued through merely maintaining the initial x-velocity throughout the elapsed duration."

In the second section fo the "Diagram the relationship" row, draw another flowchart showing that initial x-velocity $v_{x,i}$ (v-sub-x-i) contributes to final x-velocity $v_{x,f}$ (v-sub-x-f) but that constant x-acceleration a_x (a-sub-x) and accrued x-displacement (Delta x) also contribute to a quantity that contributes to final x-velocity $v_{x,f}$ (v-sub-x-f). Recite a story: "If x-velocity weren't changing, the initial x-velocity would determine (and equal) the final x-velocity, but if the x-velocity is changing with a constant x-acceleration maintained while traveling through an x-displacement, the final x-velocity would differ from the initial x-velocity."

The row labeled "Graphically present quantities" will be divided into two sections.

In the first section, write the title "v_x-t (v-sub-x-t) plot sometimes allows avoiding some algebra". Create an axis system with x-velocity v_x (v-sub-x) on the vertical axis and time t on the horizontal axis. Draw a linear plot with positive slope and positive vertical intercept. Draw two dots on the plot. Draw the corresponding tickmarks on the t axis, one at the left labeled with the initial time t_i (t-sub-i) and one at the right labeled with the final time t_f (t-sub-f). Draw the corresponding tickmarks on the v_x (v-sub-x) axis, with the lower one labeled with the initial x-velocity $v_{x.i}$ (v-sub-x-i) and the upper one labeled with the final x-velocity $v_{x,f}$ (v-sub-x-f).

From the tickmark at the left, draw an arrow upward to reach the graph. Draw a dot where the arrow meets the graph. Label this vertical arrow with the initial x-velocity $v_{x,i}$ (v-sub-x-i). From the tickmark at the right, draw an arrow upward to reach the graph. Draw a dot where this vertical arrow meets the graph. Draw an arrow from the dot at the left horizontally to the right to the vertical arrow at the right. Label this horizontal arrow with the elapsed duration (Delta t). This horizontal arrow divides into a lower portion and an upper portion the vertical arrow at the right. Label the upper portion with the change in x-velocity (Delta v-sub-x) = the constant x-acceleration a_x (a-sub-x) times the elapsed duration (Delta t). In the portion of the graph between the vertical arrows at initial time t_i (t-sub-i) and final time t_f (t-sub-f), shade the trapezoidal region between the graph and the horizontal t axis.

In the second section of the “Graphically present quantities” row, write the title, “Beginning-Middle-End chart”. In the first row of this chart, write “(t-sub-i =)”. In the second row, write in two side-by-side columns “(x-sub-i =)” and “(y-sub-i =)”. In the next row, write in two side-by-side columns “(v-sub-x-i =)” and “(v-sub-y-i =)”. In the next row, write “$[t_i, t_f]$ (interval notation: open bracket t-sub-i, t-sub-f close bracket”. In the next row, write in two side-by-side columns “(a-sub-x =)” and “(a-sub-y =)”. In the next row, write “(t-sub-f =)”. In the next row, write in two side-by-side columns “ (x-sub-f =)” and “ (y-sub-f =)”. In the next row, write in two side-by-side columns “(v-sub-x-f =)” and “(v-sub-y-f)”.

In the row labeled, “Mathematical relationship”, write (v-sub-x-i times Delta t + half a-sub-x times the square of Delta t = Delta x) and (the square of v-sub-x-i + 2 times a-sub-x times Delta x = the square of v-sub-x-f).