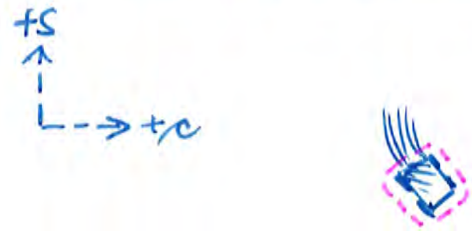
K13

Title		Magnitude of a vector in terms of its components

Ingredients

Sketch



At/Through: t

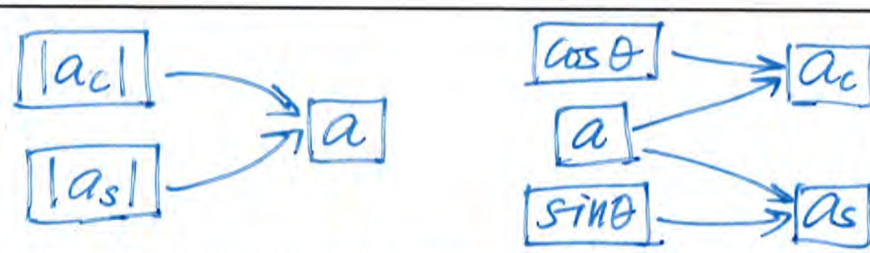
Owner: Frame; System

	Cartesian coordinates	c-component of vector $\vec{a}$	s-component of vector $\vec{a}$	angle between vector $\vec{a}$ and +c direction	magnitude of vector $\vec{a}$
Variable	(c, s)	a_c	a_s	θ	a

Giver

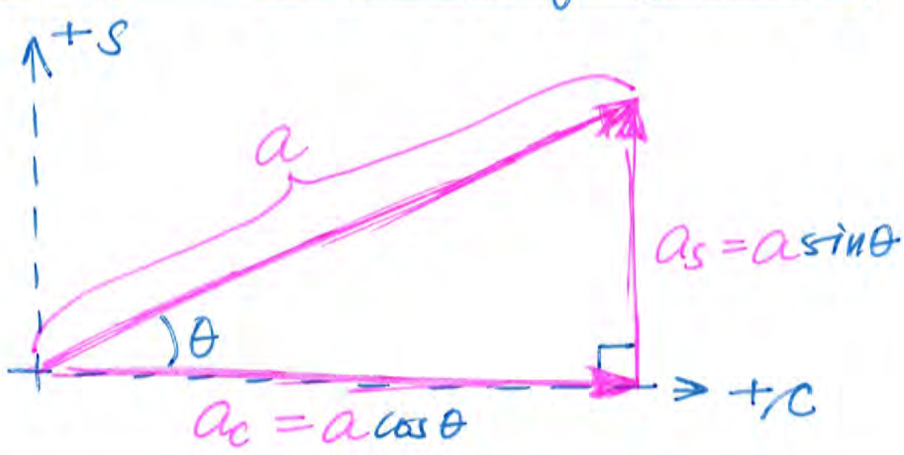
Recipe

Diagram the relationship



Graphically present quantities

Right triangle for applying trigonometry



Mathematical relationship

$$a = \sqrt{a_c^2 + a_s^2}$$

$$a_c = a\cos\theta \qquad a_s = a\sin\theta$$

Recipe number **K13**: The **title** of this recipe sheet is “**Magnitude of a vector in terms of its components**”.

The top half of this sheet consists of an “**Ingredients**” section with a row labeled “Sketch”, a row labeled “At/Through”, a row labeled “Owner”, a row labeled “Quantity”, a row labeled “Variable”, and a row labeled “Giver.” In this sheet, the row labeled “Giver” isn’t used.

For the “Sketch”, draw an overhead view showing one snapshot of a cart moving diagonally toward the lower right. Draw curved trailing motion-blur streaks or so-called “whooshies” to emphasize instantaneous motion in the snapshot. Curve the “whooshies” so that their center of curvature is toward the upper right. Draw a dashed bubble around the snapshot of the cart. Draw an arrow labeled +c to indicate that the positive-c direction happens to point to the right. From the tail of the arrow you just drew, draw another arrow labeled +s to indicate that the positive-s direction happens to point up the page.

In the rows of the “Ingredients” section other than the row for the sketch, document the following relationships, using flowchart paths, if helpful: There are two “Owners”: one is the “System”, and the other is the “Frame”. At time t, the “Frame”, meaning the collection of rulers and clocks used to make measurements and referred together as the “frame of reference”, owns the “Quantity” “Cartesian coordinates” denoted by the “Variables” in the ordered pair (c, s) (c comma s). Also at time t, the “System” owns four quantities: the “Quantity” “c-component of vector $\vec{a}$ (a with vector accent, which resembles rightward harpoon)” denoted by “Variable” a_c (a-sub-c), the “Quantity” “s-component of vector $\vec{a}$ (a with vector accent)” denoted by “Variable” a_s (a-sub-s), the “Quantity” “Angle between vector $\vec{a}$ (a with vector accent) and +c direction” denoted by “Variable” θ (theta), and the “Quantity” “Magnitude of vector $\vec{a}$ (a with vector accent)” denoted by “Variable” a (a).

The bottom half of this sheet consists of a “**Recipe**” section with a row labeled “Diagram the relationship”, a row labeled “Graphically present quantities”, and a row labeled “Mathematical relationship”.

In the row labeled, “Diagram the relationship”, draw two flowcharts. In the first flowchart, draw an arrow to indicate that the absolute value of the c-component of the vector $\vec{a}$ (a with the vector accent) a_c (a-sub-c) contributes to the magnitude of the vector $\vec{a}$ (a with vector accent) a (a). Still in the first flowchart, draw another arrow to indicate that the absolute value of the s-component of the vector $\vec{a}$ (a with vector accent) a_s (a-sub-s) also contributes to the magnitude of the vector $\vec{a}$ (a with vector accent) a (a). In the second flowchart, draw an arrow to indicate that the cosine of the angle between vector $\vec{a}$ (a with vector accent) and the +c direction θ (theta) contributes to the c-component of vector $\vec{a}$ (a with vector accent) a_c (a-sub-c). Still in the second flowchart, draw an arrow to indicate that the sine of the angle between vector $\vec{a}$ (a with vector accent) and the +c direction θ (theta) contributes to the s-component of vector $\vec{a}$ (a with vector accent) a_s (a-sub-s). Still in the second flowchart, draw two arrows to show that the magnitude of vector $\vec{a}$ (a with vector accent) a (a) contributes to both a_c (a-sub-c) and a_s (a-sub-s).

In the row labeled “Graphically present quantities”, write the title “Right triangle for applying trigonometry”. Create an axis system by drawing a dashed horizontal axis labeled +c to indicate that the positive-c direction points to the right and a dashed vertical axis labeled +s to indicate that the positive-s direction points up the page. Draw an arrow starting from the origin of the axis system and pointing diagonally toward the upper right. Label the size of this diagonal arrow using a brace labeled a (a). Using this slanted arrow as a hypotenuse, draw a right triangle with a horizontal leg and a vertical leg. Add an arrowhead to make the horizontal leg become a rightward arrow. Label this horizontal arrow with $a_c = a\cos\theta$ (a-sub-c = a cosine theta). Add an arrowhead to make the vertical leg become an upward arrow. Label this vertical arrow with $a_s = a\sin\theta$ (a-sub-s = a sine theta). Draw a perpendicular symbol in the interior right angle of the right triangle.

In the row labeled, “Mathematical relationship”, write three equations. The first equation is $a = \sqrt{a_c^2 + a_s^2}$ (a = square root of the sum of the square of a-sub-c and the square of a-sub-s). The second equation is $a_c = a\cos\theta$ (a-sub-c = a cosine theta). The third equation is $a_s = a\sin\theta$ (a-sub-s = a sine theta).