

Title	Impulse-momentum theorem		
Ingredients	Sketch		
	At/Through	$[t_i, t_f]$	
	Owner	System	
	Quantity	Net impulse	Total momentum
	Variable	$\sum \vec{J}, \sum J_x, \sum J_y$	$\sum \vec{p}_i, \sum p_{xi}, \sum p_{yi} \quad \sum \vec{p}_f, \sum p_{xf}, \sum p_{yf}$
	Giver	External environment	
Recipe	Diagram the relationship		
	Graphically present quantities	<div style="display: flex; justify-content: space-around;"> <div data-bbox="414 1218 966 1333"> <p>Impulse-momentum vector-addition diagram</p> </div> <div data-bbox="990 1218 1542 1333"> <p>x-impulse-x-momentum bar chart</p> </div> </div> <p style="text-align: center;"> $p_{1,x,i} + p_{2,x,i} + \sum J_x = p_{1,x,f} + p_{2,x,f}$ </p>	
	Mathematical relationship	$\sum \vec{p}_i + \sum \vec{J} = \sum \vec{p}_f$ $\sum p_{xi} + \sum J_x = \sum p_{xf} \quad \sum p_{yi} + \sum J_y = \sum p_{yf}$	

The top half of this sheet consists of an “**Ingredients**” section with a row labeled “Sketch”, a row labeled “At/Through”, a row labeled “Owner”, a row labeled “Quantity”, a row labeled “Variable”, and a row labeled “Giver.”

Sketch: Initial time $t\text{-sub-i}$: Two blocks are on a horizontal surface. Block 1 at the left is larger and labeled $m\text{-sub-l,1}$. Block 2, at the right, is labeled $m\text{-sub-l,2}$. Whooshies trail to the left behind block 1, indicating that block 1 is in the midst of moving toward the right. Block 2, which is momentarily at rest, is being pulled toward the right by a taut rope.

Final time $t\text{-sub-f}$: The blocks, still labeled $m\text{-sub-l,1}$ and $m\text{-sub-l,2}$ as in the initial snapshot, are still on the surface, though positioned farther to the right. The blocks are in contact and moving together toward the right, with block 1 still to the left of block 2. The whooshies trailing to the left behind block 1 are longer than in the initial snapshot, indicating a greater speed. The rope pulling block 2 toward the right is still taut.

Dashed axis arrows indicate that $+x$ points toward the right and $+y$ points up the page.

Remaining rows of Ingredients section are used for a flowchart illustrating the following:

The Owner is the System. Through the interval from initial time $t\text{-sub-i}$ to final time $t\text{-sub-f}$, the System accrues the Quantity Net impulse associated with Variables $\Sigma\text{-J-vector}$, $\Sigma\text{-J-sub-x}$, and $\Sigma\text{-J-sub-y}$ given by Giver External environment. At initial time $t\text{-sub-i}$, the System owns the Quantity Initial total momentum associated with Variables $\Sigma\text{-p-vector-sub-i}$, $\Sigma\text{-p-sub-x,i}$, and $\Sigma\text{-p-sub-y,i}$. At final time $t\text{-sub-f}$, the System owns the Quantity Final total momentum associated with Variables $\Sigma\text{-p-vector-sub-f}$, $\Sigma\text{-p-sub-x,f}$, and $\Sigma\text{-p-sub-y,f}$.

The bottom half of this sheet consists of a “**Recipe**” section with a row labeled “Diagram the relationship”, a row labeled “Graphically present quantities”, and a row labeled “Mathematical relationship”.

Diagram the relationship

A flowchart arrow shows that initial total x-momentum $\Sigma\text{-p-sub-x,i}$ contributes to final total x-momentum $\Sigma\text{-p-sub-x,f}$. Another arrow shows that net x-impulse $\Sigma\text{-J-sub-x}$ also contributes to final total x-momentum $\Sigma\text{-p-sub-x,f}$.

Graphically present quantities

Title of first section: Impulse-momentum vector-addition diagram

Draw an arrow pointed to the right labeled $p\text{-vector-sub-1,i}$. From the head of this arrow, draw a longer arrow also pointed toward the right labeled $\Sigma\text{-J-vector}$. Immediately underneath the two arrows just now drawn, draw another pair of arrows, also touching in a head-to-tail fashion, and also both pointing horizontally to the right. The arrow at the left is longer and labeled $p\text{-vector-sub-1,f}$. The arrow at the right is labeled $p\text{-vector-sub-2,f}$. The two rows of vectors are equally wide and horizontally centered together.

Title of second section: x-impulse-x-momentum bar chart

Plot $p\text{-sub-x}$ on the vertical axis. Draw a tickmark labeled 0. At the height of this tickmark, draw five horizontal segments to the right, with one segment each above each of the terms of the labeling equation $p\text{-sub-1,x,i} + p\text{-sub-2,x,i} + \Sigma\text{-J-sub-x} = p\text{-sub-1,x,f} + p\text{-sub-2,x,f}$. From the segment above the term $p\text{-sub-1,x,i}$, extend upward a short, shaded rectangular bar. Write the number 0 above the segment above the term $p\text{-sub-2,x,i}$. From the segment above the term $\Sigma\text{-J-sub-x}$, extend upward a tall, shaded rectangular bar. From the segment above the term $p\text{-sub-1,x,f}$, extend upward an even taller shaded rectangular bar. From the segment above the term $p\text{-sub-2,x,f}$, extend upward a short, shaded rectangular bar. Scale the heights of the bars so that the sum of the heights of the bars for the terms $p\text{-sub-1,x,i}$ and $\Sigma\text{-J-sub-x}$ equals the sum of the heights of the bars for the terms $p\text{-sub-1,x,f}$ and $p\text{-sub-2,x,f}$.

Mathematical relationship

$$\Sigma\text{-p-vector-sub-i} + \Sigma\text{-J-vector} = \Sigma\text{-p-vector-sub-f}$$

$$\Sigma\text{-p-sub-x,i} + \Sigma\text{-J-sub-x} = \Sigma\text{-p-sub-x,f}$$

$$\Sigma\text{-p-sub-y,i} + \Sigma\text{-J-sub-y} = \Sigma\text{-p-sub-y,f}$$