E1

Title: Work performed by a force

Ingredients

Sketch	
At/Through	t_i t_f t $[t_i, t_f]$
Owner	Point-like; System
Quantity	Path length so far traveled; Instantaneous velocity; Force; Work
Variable	l_i l_f l $\vec{v}$ $\vec{F}, F_{\parallel}, F_{\perp}$ W_F
Giver	external agents

Recipe

Diagram the relationship	$F_{\parallel}$, l → W_F
Graphically present quantities	Partial force diagram; On $F_{\parallel}$-l plot: Signed area under curve; Work bar chart
Mathematical relationship	Signed area between $F_{\parallel}$-l plot and l axis $= W_F$

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The top half of this sheet consists of an "**Ingredients**" section with a row labeled "Sketch", a row labeled "At/Through", a row labeled "Owner", a row labeled "Quantity", a row labeled "Variable", and a row labeled "Giver."

Sketch: Three snapshots of a sphere, not all at the same height, are arranged from left to right with trailing whooshies showing that the sphere is in the midst of moving in each snapshot. In each snapshot, draw a dot on the sphere. The dot doesn't need to be on the same point on the sphere in the snapshots. For the middle snapshot, place the dot at the bottom-left limb of the sphere. Draw a dashed, curved path that passes through the three dots, with occasional arrowheads on the path indicating a general movement from left to right. Draw the path so that, in at least one snapshot, the trailing whooshies indicate an instantaneous velocity of the sphere that is not in the same direction as the direction of motion along the curved path. This shows that the path taken by the point of interest on the sphere is conceptually distinct from the path taken by the sphere itself. Surround the sphere in the middle snapshot with the dashed system bubble. Draw a hand using its index finger to push toward the upper right on the dot in the middle snapshot.

Remaining rows of Ingredients section are used for a flowchart illustrating the following:
There are two Owners: A point-like external agent and the System. At initial time t-sub-i, the point-like external agent owns the Quantity Initial path length so far traveled denoted by the Variable cursive-l-sub-i. At final time t-sub-f, the point-like external agent owns the Quantity Final path length so far traveled denoted by the Variable cursive-l-sub-f. At time t, the point-like external agent owns the Quantity Path length so far traveled denoted by the Variable cursive-l. At time t, the point-like external agent also owns the Quantity Instantaneous velocity denoted by the Variable v-vector. At time t, the System receives the Quantity Force associated with Variables F-vector, F-sub-parallel, and F-perpendicular given by Giver point-like external agent. Through the interval from initial time t-sub-i to final time t-sub-f, the System receives the Quantity Work denoted by the Variable W-sub-F given by Giver point-like external agent.

The bottom half of this sheet consists of a "**Recipe**" section with a row labeled "Diagram the relationship", a row labeled "Graphically present quantities", and a row labeled "Mathematical relationship".

Diagram the relationship
A flowchart arrow shows that the component of the force parallel to the instantaneous velocity of the point-like external agent contributes to the work performed by the force W-sub-F. Another arrow shows that path length so far traveled cursive-l also contributes to the work W-sub-F.

Graphically present quantities
Title of first section: Partial force diagram
Draw a dot representing the point of contact between the index finger and the sphere in the middle snapshot in the sketch in the Ingredients section. Through the dot you've just drawn, draw a dashed curve representing a magnified view of the portion of the dashed curve from the sketch in the Ingredients section in the neighborhood of the dot from the middle snapshot. Through the dot, draw a dashed axis pointing upward and to the right, tangent to the curved path at the dot. Label this axis +parallel. Draw a second dashed axis perpendicular to the first. Label this axis +perpendicular. Draw a force arrow, labeled F-sub-F-on-sys, originating from the dot and pointing upward and toward the right, but not exactly in the same direction as the +parallel axis. Using this force arrow as a hypotenuse, complete a diagram of a right-triangle with a leg parallel to the +parallel axis labeled F-sub-H-on-sys,parallel and a leg parallel to the +perpendicular axis labeled F-sub-H-on-sys,perpendicular. Draw an arrowhead on each of these legs so that, together, they are touching in a head-to-tail fashion showing that they vectorially add to give a resultant vector equivalent to the force vector labeled F-sub-H-on-sys. Draw a right-angle symbol at the right interior angle of the right triangle.

Title of second section: On F-sub-parallel-cursive-l plot: Signed area under curve
Plot F-sub-parallel along the vertical axis. Plot time cursive-l along the horizontal axis. Draw a generic graph of a function with some positive and negative values. Draw tickmarks for initial path length so far traveled cursive-l-sub-i and final path length so far traveled cursive-l-sub-f on the path-length axis so that the interval between

these path lengths includes path lengths for which the parallel-component of the force is positive and path lengths for which the parallel component of the force is negative. Draw vertical segments extending from the tickmarks on the path-length axis to the graph. Shade the region between the F-sub-parallel-cursive-l plot and the cursive-l axis between initial path length so far traveled cursive-l-sub-i and final path length so far traveled cursive-l-sub-f. Within each shaded region above the path-length axis, write a positive sign. Within each shaded region below the path-length axis, write a negative sign. Draw arrows originating from each such shaded region and pointing toward a caption box for the work performed by the force, W-sub-F.

Title of third section: Work bar chart
Plot W on the vertical axis. Draw a tickmark labeled 0. At the height of this tickmark, draw a horizontal segment to the right, labeled underneath as W-sub-F. From and extending upward from this labeled segment, draw a shaded rectangular bar.

Mathematical relationship
Signed area between F-sub-parallel-cursive-l plot and l axis = W-sub-F