

Title

Generalized work-energy principle

E6

Ingredients

Sketch



At/Through

t_i

t_f

$[t_i, t_f]$

Owner

System

Quantity

Total energy

Total work

Variable

$$\sum K_i + \sum U_{G,i} + \sum U_{S,i} + \sum U_{INT,i}$$

$$\sum K_f + \sum U_{G,f} + \sum U_{S,f} + \sum U_{INT,f}$$

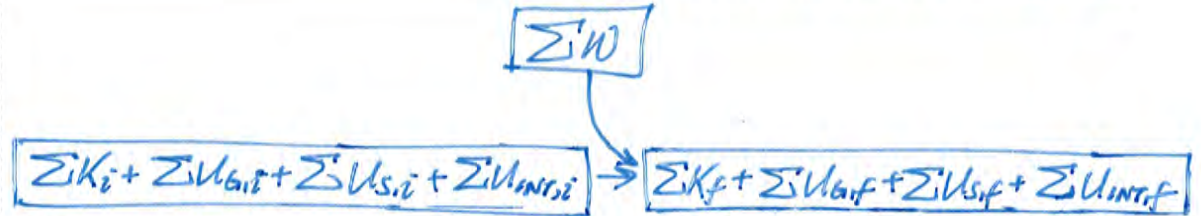
$$\sum W$$

Giver

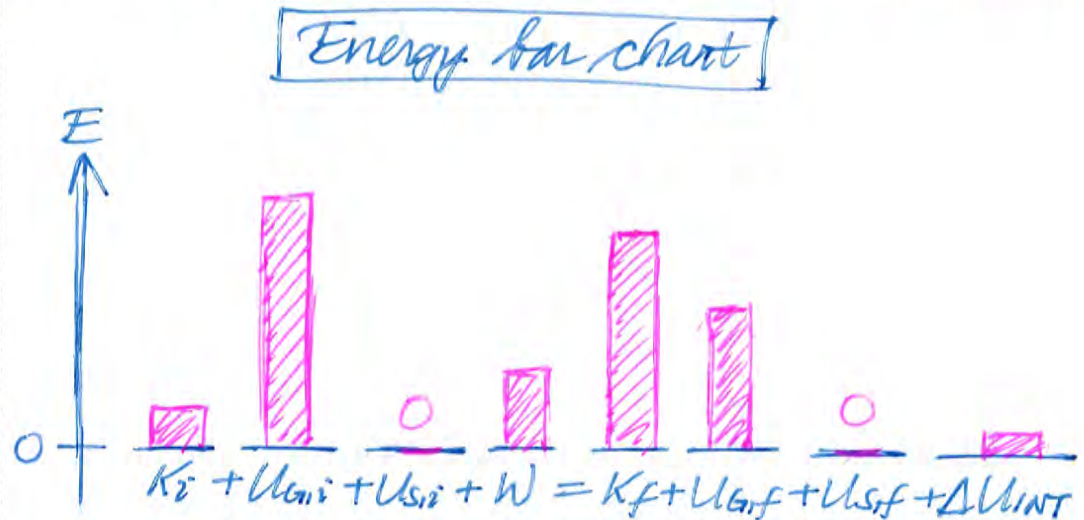
External environment

Recipe

Diagram the relationship



Graphically present quantities



Mathematical relationship

$$\sum K_i + \sum U_{G,i} + \sum U_{S,i} + \sum U_{INT,i} + \sum W = \sum K_f + \sum U_{G,f} + \sum U_{S,f} + \sum U_{INT,f}$$

Shorthand: $K_i + U_{G,i} + U_{S,i} + W = K_f + U_{G,f} + U_{S,f} + \Delta U_{INT}$

The top half of this sheet consists of an “**Ingredients**” section with a row labeled “Sketch”, a row labeled “At/Through”, a row labeled “Owner”, a row labeled “Quantity”, a row labeled “Variable”, and a row labeled “Giver.”

Sketch: A roller coaster track with peaks and valleys is on a firm, horizontal surface. Under the surface is the label, “Earth”. A cart on the roller-coaster track is shown in an earlier snapshot at a higher height, more to the left and a later snapshot at a lower height, more to the right. Trailing whooshies indicate that the cart is moving in both snapshots. The whooshies are longer in the second snapshot. A dashed system bubble surrounds the cart, the roller-coaster track, the surface, and the label, “Earth”. Outside of the dashed bubble, a cartoon of a cloud blows air along the track. A vertical dashed axis arrow shows that +h points up the page. A tickmark on the vertical axis indicates that the height $h = 0$ at the surface of the Earth. The placement of the tickmark for 0 on the vertical axis is arbitrary. Choosing the surface of the Earth to have a height of 0 is merely a common choice.

Remaining rows of Ingredients section are used for a flowchart illustrating the following:

The Owner is the System. At initial time $t\text{-sub-}i$, the System owns the Quantity Initial total energy denoted by the Variable expression $\Sigma K\text{-sub-}i + \Sigma U\text{-sub-}G,i + \Sigma U\text{-sub-}S,i + \Sigma U\text{-sub-}INT,i$. At final time $t\text{-sub-}f$, the System owns the Quantity Final total energy $\Sigma K\text{-sub-}f + \Sigma U\text{-sub-}G,f + \Sigma U\text{-sub-}S,f + \Sigma U\text{-sub-}INT,f$. Through the course of the interval from initial time $t\text{-sub-}i$ to final time $t\text{-sub-}f$, the System receives the Quantity Total Work denoted by the Variable ΣW given by Giver External environment.

The bottom half of this sheet consists of a “**Recipe**” section with a row labeled “Diagram the relationship”, a row labeled “Graphically present quantities”, and a row labeled “Mathematical relationship”.

Diagram the relationship

A flowchart arrow shows that Initial total energy $\Sigma K\text{-sub-}i + \Sigma U\text{-sub-}G,i + \Sigma U\text{-sub-}S,i + \Sigma U\text{-sub-}INT,i$ contributes to Final total energy $\Sigma K\text{-sub-}f + \Sigma U\text{-sub-}G,f + \Sigma U\text{-sub-}S,f + \Sigma U\text{-sub-}INT,f$. Another arrow shows that Total work ΣW also contributes to Final total energy $\Sigma K\text{-sub-}f + \Sigma U\text{-sub-}G,f + \Sigma U\text{-sub-}S,f + \Sigma U\text{-sub-}INT,f$.

Graphically present quantities

Energy bar chart

Plot E on the vertical axis. Draw a tickmark labeled 0. At the height of this tickmark, draw eight horizontal segments to the right, spaced apart so that each horizontal segment is labeled underneath by one of the terms of the equation $K\text{-sub-}i + U\text{-sub-}G,i + U\text{-sub-}S,i + W = K\text{-sub-}f + U\text{-sub-}G,f + U\text{-sub-}S,f + \Delta U\text{-sub-}INT$. Fill in a concrete example bar chart. Write the number 0 above the terms $U\text{-sub-}S,i$ and $U\text{-sub-}S,f$. This is consistent with the Sketch from the “Ingredients” section. The Sketch doesn’t show any springs, so the System doesn’t own Spring potential energy. From the horizontal segment labeled $K\text{-sub-}i$, extend upward a short, shaded rectangular bar representing the relatively small initial kinetic energy of the cart. From the horizontal segment labeled $U\text{-sub-}G,i$, extend upward a tall, shaded rectangular bar representing the relatively high gravitational potential energy of the Earth and cart. From the horizontal segment labeled W , extend upward a modestly tall, shaded rectangular bar to represent the positive work done by the air blowing against the cart as the cart travels from the initial snapshot to the final snapshot in the Sketch in the “Ingredients” section. From the horizontal segment labeled $K\text{-sub-}f$, extend upward a rather tall, shaded rectangular bar representing the relatively high final kinetic energy of the cart. From the horizontal segment labeled $U\text{-sub-}G,f$, extend upward a modestly tall, shaded rectangular bar representing the decreased gravitational potential energy of the Earth and cart in the final snapshot. From the horizontal segment labeled $\Delta U\text{-sub-}INT$, extend upward a short, shaded rectangular bar representing a small increase in the internal energy of the System. Adjust the heights of bars in the bar chart, if needed, to make sure that the sum of the graphically represented values of the first four terms of the labeling equation equals the sum of the graphically represented values of the last four terms of the labeling equation.

Mathematical relationship

$\Sigma K\text{-sub-}i + \Sigma U\text{-sub-}G,i + \Sigma U\text{-sub-}S,i + \Sigma U\text{-sub-}INT,i + \Sigma W = \Sigma K\text{-sub-}f + \Sigma U\text{-sub-}G,f + \Sigma U\text{-sub-}S,f + \Sigma U\text{-sub-}INT,f$

Shorthand: $K\text{-sub-}i + U\text{-sub-}G,i + U\text{-sub-}S,i + W = K\text{-sub-}f + U\text{-sub-}G,f + U\text{-sub-}S,f + \Delta U\text{-sub-}INT$