Cognitive Steps for Learning, Applying, and Explaining

PHYSICS

Without Using Talent

Is this happening to you?
"I understood what I got wrong on the FRQs last time, but I still don't know how to write explanations."

You don't learn to thread a needle by looking at embroidery. Reading answer keys is different from being coached to follow a cognitive method to create explanations.

A five-day bootcamp

you can use **even if you're not on your school's forensics team** or **taking AP® Seminar!**

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1. Day 1: Obtain knowledge (30 minutes)

We can choose from different methods for obtaining knowledge.

I want to develop this kind of knowledge	I can use these methods in a conventional didactic course	I can also use these methods in an inquiry-based course
Definition of a quantity	Careful reading (Day 3) Structured note-taking (Day 2)	Experimentation (PITch, Day 1)
Relationship among quantities	Careful reading (Day 3) Mathematical derivation (SiQuENC, Day 4) Structured note-taking (Day 2)	Experimentation (PITch, Day 1)

Get PITCh steps (5 minutes)

Let's start with an example of experimentation. Print out the table below.

Step	Instruction
P	State a **<u>P</u>urpose** "I want to define a quantity to represent …" "I want to write an equation or inequality that models a relationship among …"
I	Look at a small set of **<u>I</u>nitial experiments**
T	Propose at least one **candidate (<u>T</u>entative) mathematical tool** "I propose to define … to refer to the expression …" "I propose that a manner in which the quantities … are related is …"
Ch	Carry out at least one additional experiment to **<u>Ch</u>allenge** candidate(s)

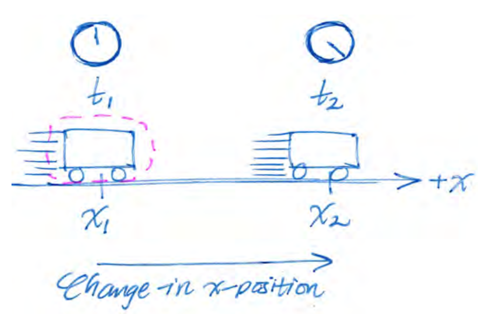
Use PITCh steps (25 minutes)

Step	Instruction
P	State a **<u>P</u>urpose** "I want to define a quantity to represent …" "I want to write an equation or inequality that models a relationship among …"

Say we want to define a quantity to represent some sort of sense and/or extent of change in x-position.

Step	Instruction
I	Look at a small set of **<u>I</u>nitial experiments**

Let the system of interest be a cart moving along the x-axis from a first x-position x_1 in a first snapshot at time t_1 to a second x-position x_2 in a second snapshot at time t_2. An arrow labeled "Change in x-position" pointing from the first x-position to the second x-position emphasizes that these x-positions are different.



Step	Instruction
T	Propose at least one **candidate (<u>T</u>entative) mathematical tool** "I propose to define … to refer to the expression …" "I propose that a manner in which the quantities … are related is …"

We want to mathematically represent some sort of sense and/or extent of change in x-position. The initial experiment draws attention to the different locations of the first and second x-positions on the x-axis. In algebra, we have multiple tools for comparing quantities. One example is a difference. Another example is a ratio. Let the difference $P = x_2 - x_1$ and the ratio $Q = \frac{x_2}{x_1}$ be candidates for expressions we deem the change in x-position.

Step	Instruction
Ch	Carry out at least one additional experiment to **Challenge** candidate(s)

How well do our candidate expressions P and Q represent the sense and extent of the change in x-position in a few more experiments?

	Raw data		Key features to compare with		Candidate expressions	
Experiment	x_1	x_2	**Sense (direction) of change in x-position**	**Extent of change in x-position**	$P = x_2 - x_1$	$Q = \frac{x_2}{x_1}$
1.	5 m	10 m	+x direction	5 m	+5 m	+2
2.	0 m	5 m	+x direction	5 m	+5 m	Undefined
3.	−2 m	−5 m	−x direction	3 m	−3 m	+5/2
4.	2 m	2 m	Undefined	0 m	0 m	+1

The difference $P = x_2 - x_1$ expresses both the sense and extent of the change in the x-position in the four experiments above. When the sense (direction) of the change in the x-position is the +x direction, the sign of the difference $P = x_2 - x_1$ is positive. When the direction of the change in x-position is the −x direction, the sign of the difference $P = x_2 - x_1$ is negative. When the direction of the change in x-position is undefined, the sign of the difference $P = x_2 - x_1$ is neither positive nor negative. The extent of the change in x-position and the absolute value of the difference $P = x_2 - x_1$ always match.

The ratio $Q = \frac{x_2}{x_1}$ doesn't do as good of a job at expressing the sense and extent of the change in x-position. We can't determine the direction of the change in the x-position from the sign of the ratio $Q = \frac{x_2}{x_1}$, which happens to be positive in experiments 1, 3, and 4 even though none of these experiments have matching directions of change in x-position. We can't determine the extent of the change in x-position from the absolute value of the ratio $Q = \frac{x_2}{x_1}$. The extent of the change in x-position is identical in experiments 1 and 2, but the ratio $Q = \frac{x_2}{x_1}$ doesn't have the same absolute value for these two experiments. The ratio $Q = \frac{x_2}{x_1}$ is undefined in experiment 2! I'd prefer to avoid having a quantity that can become undefined in an ordinary situation.

The difference $P = x_2 - x_1$ remains an attractive possibility for a quantity to use to represent the change in the x-position. We reject ratio $Q = \frac{x_2}{x_1}$ for this purpose. Newly confident that we'd like to work with the difference $P = x_2 - x_1$, let's give P a special symbol, Δx, and a special name, "x-displacement". We write

$$\Delta x = x_2 - x_1$$

or, if we prefer to avoid a subtraction symbol,

$$x_1 + \Delta x = x_2$$

To create a "blank template" with room for custom subscripts, change the subscripted numerals to blank spaces:

$$x_{\_} + \Delta x = x_{\_}$$

An investigation similar to the one for x-displacement can lead to a similar defined quantity, presented on Day 2, for the change in time.

2. Day 2: Take notes for elapsed duration, x-displacement (30 minutes)

Allowed knowledge

Notion of a system
We draw a dashed bubble to indicate that we are regarding the enclosed object(s) as the **system** of interest.

Definitions of elapsed duration and x-displacement

Suppose we depict a system in two snapshots from the same story.

The **elapsed duration** is the change in time Δt added to the first snapshot's time $t_{\_}$ to get the second snapshot's time $t_{\_\_}$:

$$t_{\_} + \Delta t = t_{\_\_}$$

The **x-displacement** is the change in x-position Δx added to the system's first x-position $x_{\_}$ to get the system's second x-position $x_{\_\_}$:

$$x_{\_} + \Delta x = x_{\_\_}$$

Structure your notes

TIP: Take structured notes
Organize notes for a given mathematical idea into sections based on format (**graphical**, **verbal**, and **algebraic**) and based on components of conditional statements (**conditions** that must be met, **conclusions and algebraic relationships** that can be used when conditions are met).

Fold an 8.5x11" sheet into four panels of equal size. Fill in the sheet using the model below.

Elapsed duration and x-displacement

1. Is scenario eligible for analysis with this sheet?

1.a. Inspect sketch

1.b. Document qualifications

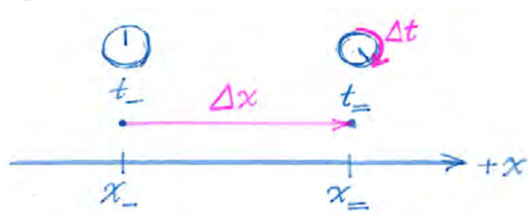
System (in bubble):	
+x direction (draw arrow):	
Label for _st snapshot:	
Label for _nd snapshot:	

2. If scenario is eligible, continue using this sheet.

2.a. Draw a diagram

Elapsed duration curve – directed arc traces sweep of clock hand tip

Displacement vector – arrow from _st position dot to _nd position dot



2.b. Fill in and compute values in a form

_st time	+	Elapsed duration	=	_nd time
$t_{\_}$	+	Δt	=	$t_{\_\_}$
	+		=	

_st x-position	+	x-displacement	=	_nd x-position
$x_{\_}$	+	Δx	=	$x_{\_\_}$
	+		=	

3. Day 3: Carefully reading a problem (1 hr 45 min – 2 hr 30 min)

Copy a practice problem (5 minutes)

TIP: Practice on paper

Start each problem by copying the problem statement onto the same page where you'll start showing your work. You can copy the problem statement by hand or you can tape a photocopy of the problem statement into your homework. Don't just look at a problem statement on a screen.

At the top of a fresh page of homework, copy the problem statement for practice problem "Alice" (below).

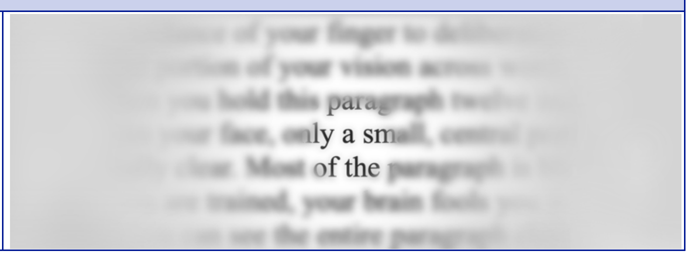
Practice Problem: Alice. In an experimental trial, Alice walks along the x-axis from one location to another. In a second trial, the x-positions of Alice's starting and ending locations are both increased by the same number of meters. How does the x-displacement in Alice's second trial compare with the x-displacement in her first trial? Choose an option and explain your reasoning.

A. Alice's 2^{nd} x-displacement is greater than her 1^{st} x-displacement.
B. Alice's 2^{nd} x-displacement equals her 1^{st} x-displacement.
C. Alice's 2^{nd} x-displacement is less than her 1^{st} x-displacement.
D. Cannot be determined.

Get the reading guide (10 minutes)

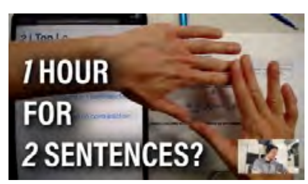
TIP: Point as you read aloud 指差喚呼

Use the guidance of your finger to deliberately drag the central portion of your vision across words as you read. When you hold this paragraph twelve inches away from your face, only a small, central portion of it is actually clear. Most of the paragraph is blurry. Unless you are trained, your brain fools you into thinking you can see the entire paragraph clearly.



Print out the **reading guide** below **or open** an electronic version at davidliao.com/read.php.

Optional (adds 60 minutes): **Watch** a demo of using a **reading guide** to read mathematical English at youtu.be/KMP4T4cyRuc. (This video uses a previous version of the reading guide).



Step	Instruction
0	**Welcome!** These steps help you to understand problems that are too complicated to understand at first glance. **Draw** an erasable **dot** immediately before the first word of the problem statement or reading assignment. Start reading from the dot. Stop when you've read just enough words so that you've read **at most** a **noun and** a **verb**, a **prepositional phrase**, or another equivalently **small amount** of information (roughly 1-7 words). **Draw a second** erasable **dot** immediately after the last word you just read. **Go to 1**
1	**Read** the words **between the dots**. **Go to 2**
2	Methodically **scan** your drawing(s) and table(s) **for** features that **contradict** the words between the dots. Found **no contradiction**. **Go to 5** **Found** ≥ 1 **contradiction**. **Go to 3**
3	**Edit/update** one feature of your drawing(s) or table(s) that contradicts the words between the dots. **Go to 4**
4	Did your edit involve **adding information** from the words between the dots to your drawing(s) or table(s)**?** **No,** I merely **repaired** an existing feature of my drawing(s) or table(s). **Go to 1** **Yes,** when I edited my drawing(s) or table(s), I **added** a feature to my drawing(s) or table(s). **Go to 8**
5	**Read** the words **between the dots**. **Go to 6**
6	Methodically **scan** your drawing(s) and table(s) to **check** whether **all meaning** in the words between the dots has been **represented** somewhere in your drawing(s) and table(s). Is there **information** that is in the words between the dots but **missing** in your drawing(s)/table(s)**?** **No information** is **missing**. **Go to 9** **Some information** is **missing**. **Go to 7**
7	**Add** a feature to the drawing(s) or table(s) to represent one piece of information not yet represented from the words between the dots. **Go to 8**
8	**<u>Underline</u>** the word(s) and/or symbol(s) between the dots that correspond to the feature you just added to your drawing(s) or table(s). **Go to 1**
9	Lightly <u>underline</u> any portion(s) of the words between the dots that remain to be underlined. **Go to 10**
10	Are there **words left** to parse in the problem statement or reading assignment**?** **Yes,** words remain to be parsed. **Go to 11** **No,** all words have been parsed. **Go to 12**
11	**Erase** the **dot next to the first word** between the dots. Start reading from the remaining dot. Stop when you've read just enough words so that you've read **at most** a **noun** and a **verb**, a **prepositional phrase**, or another equivalently **small amount** of information (roughly 1-7 words). **Draw a new erasable dot** immediately after the last word you just read. **Go to 1**
12	**Congratulations!** You have finished initial digestion of the problem statement or reading assignment.

Get started with the reading guide (15 minutes)

Read step 0.

Step	Instruction
0	**Welcome!** These steps help you to understand problems that are too complicated to understand at first glance. **Draw** an erasable **dot** immediately before the first word of the problem statement or reading assignment. Start reading from the dot. Stop when you've read just enough words so that you've read **at most** a **noun and** a **verb**, a **prepositional phrase**, or another equivalently **small amount** of information (roughly 1-7 words). **Draw a second** erasable **dot** immediately after the last word you just read. **Go to 1**

In the problem statement, draw a dot just before the word "In" in the phrase "In an experimental trial,". Use your finger to point to the "In". Read aloud the word "In". Think, "I've only read a preposition, not a noun and a verb, not a prepositional phrase, and not another phrase with an equivalent amount of information. I haven't read enough information to draw a new dot, so I'll keep reading." Move your finger to the "an". Read aloud the "an". Think, "I've only read a preposition and an indefinite article, not a noun and a verb, not a prepositional phrase, and not another phrase with an equivalent amount of information. I haven't read enough information to draw a new dot, so I'll keep reading." Continue in this way until you've just finished reading the word, "trial," for the first time. At this point, think, "I finally just now barely read a prepositional phrase." Draw a second dot just after the word you just read, "trial,". (As you become fluent, you might skip explicitly narrating more and more of the "I've only read … not … so I'll keep reading." statements).

Follow the instruction "Go to 1" by going to step 1. Read step 1.

Step	Instruction
1	**Read** the words **between the dots**. **Go to 2**

Use your finger to point to the "In". Read aloud the word, "In". Move your finger to the word "an". Read aloud the word "an". Move your finger to the syllable "ex". Read aloud the syllable "ex". Move your finger to the syllable "per". Read aloud the syllable "per". Continue dragging your finger and reading aloud the corresponding syllable, one syllable at a time, at the slowest speed that sounds like meaningful speech. Make sure your finger is never ahead of your speech by more than about one syllable, *at most*. Stop when you've finished reading aloud all the words between the dots, "In an experimental trial,".

Read step 2.

Step	Instruction
2	Methodically **scan** your drawing(s) and table(s) **for** features that **contradict** the words between the dots. Found **no contradiction**. **Go to 5** **Found** ≥ 1 **contradiction**. **Go to 3**

Starting just underneath the problem statement, take about three seconds to drag your **finger across** the **page** from left to right, **wiggling** your finger **up** 2 inches **and down** 2 inches several times along the way. Then, repeat the dragging and wiggling of your finger across an adjacent band of the page below the region you just scanned, and so on, until you've visually scanned all of your workspace for drawings and tables for the problem.

Your workspace is still blank, so there is nothing in your workspace that contradicts the words between the dots, “In an experimental trial,”, and you should choose “Found **no contradiction**”, which means reading step 5.

Step	Instruction
5	**Read** the words **between the dots**. **Go to 6**

The dots haven’t moved. They still enclose the phrase, “In an experimental trial,”, so read aloud “In an experimental trial,”, pointing with your finger and deliberately attending to each syllable, as described before.

Read step 6.

Step	Instruction
6	Methodically **scan** your drawing(s) and table(s) to **check** whether **all meaning** in the words between the dots has been **represented** somewhere in your drawing(s) and table(s). Is there **information** that is in the words between the dots but **missing** in your drawing(s)/table(s)**?** **No information** is **missing**. **Go to 9** **Some information** is **missing**. **Go to 7**

Starting just underneath the problem statement, take about three seconds to drag your **finger across** the **page** from left to right, **wiggling** your finger **up** 2 inches **and down** 2 inches several times along the way. Then, repeat the dragging and wiggling of your finger across an adjacent band of the page below the region you just scanned, and so on, until you’ve visually scanned all of your workspace for drawings and tables for the problem.

At this point, your workspace is still blank, so all the information in the words between the dots, “In an experimental trial,”, is missing from your workspace. Choose “**Some information is missing**.”, which means reading step 7:

Step	Instruction
7	**Add** a feature to the drawing(s) or table(s) to represent one piece of information not yet represented from the words between the dots. **Go to 8**

One way to indicate that there is an experimental trial at all is to write a label called “Trial”.

Then, read step 8.

Step	Instruction
8	<u>**Underline**</u> the word(s) and/or symbol(s) between the dots that correspond to the feature you just added to your drawing(s) or table(s). **Go to 1**

In the problem statement, underline the whole phrase “In an experimental trial,”.

Then, read step 1.

Step	Instruction
1	**Read** the words **between the dots**. **Go to 2**

The dots still haven't moved. They still enclose the phrase, "In an experimental trial,", so read aloud "In an experimental trial,", pointing with your finger and deliberately attending to each syllable, as described before.

Then, read step 2.

Step	Instruction
2	Methodically **scan** your drawing(s) and table(s) **for** features that **contradict** the words between the dots. Found **no contradiction**. **Go to 5** **Found** ≥ 1 **contradiction**. **Go to 3**

Starting just underneath the problem statement, take about three seconds to drag your **finger across** the **page** from left to right, **wiggling** your finger **up** 2 inches **and down** 2 inches several times along the way. Then, repeat the dragging and wiggling of your finger across an adjacent band of the page below the region you just scanned, and so on, until you've visually scanned all of your workspace for drawings and tables for the problem.

At this point, your workspace contains merely a label that reads, "Trial". This label indicates the existence of a trial, which doesn't contradict the words between the dots, "In an experimental trial,", which also indicate that there's a trial. So, choose "Found **no contradiction**." and read step 5.

Step	Instruction
5	**Read** the words **between the dots**. **Go to 6**

The dots still haven't moved. They still enclose the phrase, "In an experimental trial,", so read aloud "In an experimental trial,", pointing with your finger and deliberately attending to each syllable, as described before.

Read step 6.

Step	Instruction
6	Methodically **scan** your drawing(s) and table(s) to **check** whether **all meaning** in the words between the dots has been **represented** somewhere in your drawing(s) and table(s). Is there **information** that is in the words between the dots but **missing** in your drawing(s)/table(s)**?** **No information** is **missing**. **Go to 9** **Some information** is **missing**. **Go to 7**

Starting just underneath the problem statement, take about three seconds to drag your **finger across** the **page** from left to right, **wiggling** your finger **up** 2 inches **and down** 2 inches several times along the way. Then, repeat the dragging and wiggling of your finger across an adjacent band of the page below the region you just scanned, and so on, until you've visually scanned all of your workspace for drawings and tables for the problem.

At this point, your workspace contains merely a label that reads, "Trial", so the information in the words between the dots, "In an experimental trial,", is represented in your workspace. Choose "**No information** is **missing**.", which means reading step 9.

Step	Instruction
9	Lightly underline any portion(s) of the words between the dots that remain to be underlined. **Go to 10**

Because you previously underlined the entire phrase "In an experimental trial,", there are no words left between the dots to underline. (As you use the reading guide for homework, you'll occasionally encounter situations where step 9 makes you underline something).

Read step 10.

Step	Instruction
10	Are there **words left** to parse in the problem statement or reading assignment**?** **Yes,** words remain to be parsed. **Go to 11** **No,** all words have been parsed. **Go to 12**

Look at the problem statement. After the words between the dots, "In an experimental trial,", there are a bunch of words, starting with "Alice walks", that haven't been read and underlined. So, choose "**Yes,** words remain to be parsed." and read step 11.

Step	Instruction
11	**Erase** the **dot next to the first word** between the dots. Start reading from remaining dot. Stop when you've read just enough words so that you've read **at most** a **noun** and a **verb**, a **prepositional phrase**, or another equivalently **small amount** of information (roughly 1-7 words). **Draw a new erasable dot** immediately after the last word you just read. **Go to 1**

Erase the dot just in front of the "In" of "In an experimental trial,". Use your finger to point at the word "Alice". Read the word "Alice". Think, "Hmm, I've only read a noun. I can still read more." Move your finger to the word "walks". Read the word "walks". Think, "Now I've read both a noun, 'Alice', and a verb, 'walks', so I'll stop." Draw a dot just after the "walks" of "Alice walks".

Then, read step 1.

Step	Instruction
1	**Read** the words **between the dots**. **Go to 2**

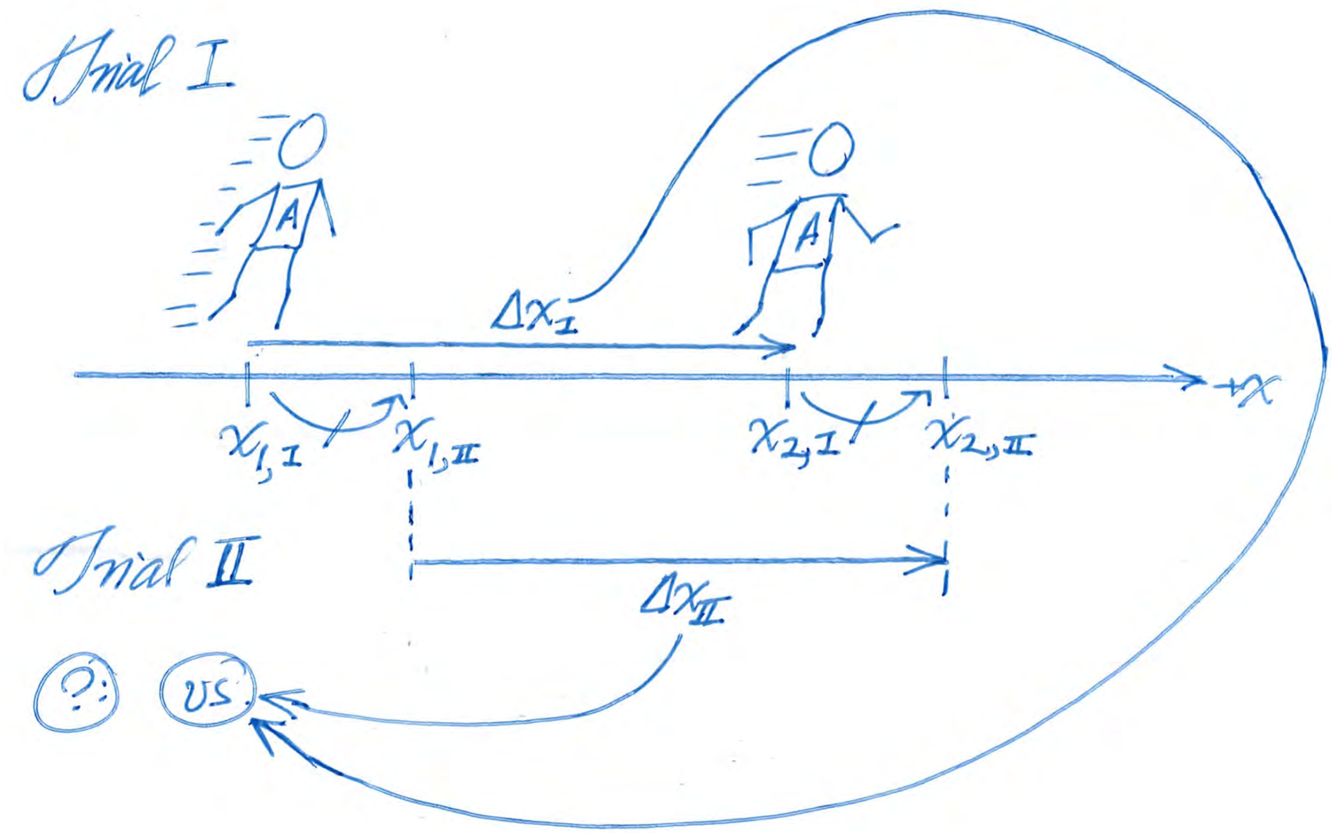
Use your finger to point to the syllable "Al" of "Alice". Read aloud the syllable, "Al". Continue dragging your finger and reading aloud the corresponding syllable, one syllable at a time, at the slowest speed that sounds like meaningful speech. Make sure your finger is never ahead of your speech by more than about one syllable, *at most*. Stop when you've just finished reading "Alice walks".

Continue using the reading guide (45 minutes)

Continue using the reading guide until you have digested the problem statement up through the question mark in "… the x-displacement in her first trial?" If you're working in a pair, have one person announce the instructions while the other person carries out the instructions for a few steps. Switch roles every now and then.

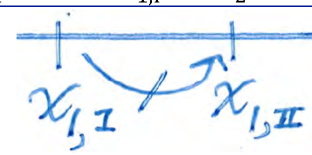
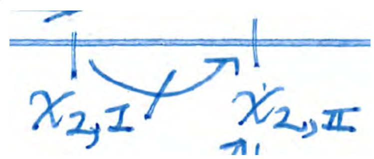
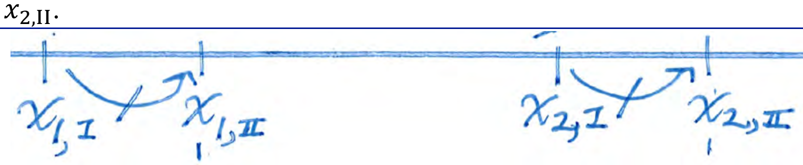
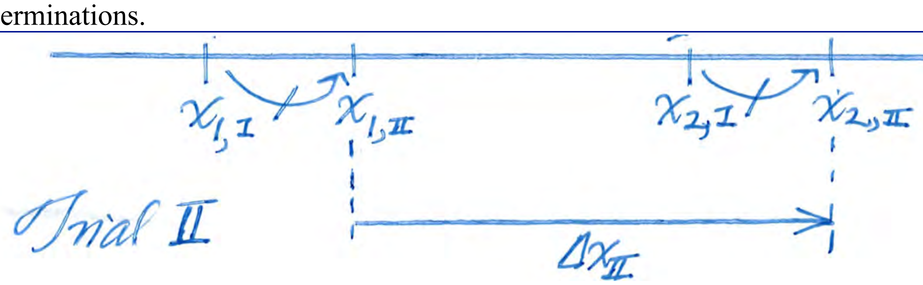
Check your reading against a key (30 minutes)

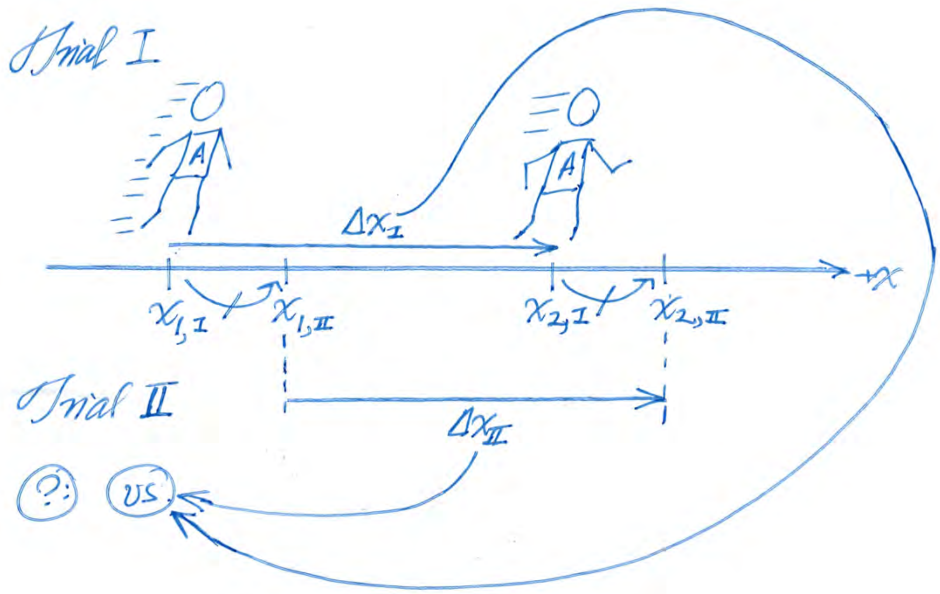
Once you've completed the reading exercise described in the previous section, check whether your workspace conveys the same meaning and the same amount of detail as in the example below.



The table below indicates when each portion of the diagram was drawn as the problem statement was read phrase by phrase.

	Stop after phrase	Draw
1.	In an experimental trial,	 "Trial" label at top-left corner.
2.	Alice walks	 Stick figure of Alice near "Trial" label. Trailing whooshies indicate Alice is in the midst of moving.
3.	along the x-axis	 Horizontal axis drawn under Alice, as though a street, labeled +x at arrowhead end. For this example, the x axis happens to point from left to right, indicating that the +x direction is toward the right.
4.	from one location	 Tickmark on x-axis directly under stick figure of Alice. Tickmark is labeled x_1.

5.	to another.	 Second copy of stick figure of Alice at another location along the x-axis, accompanied by a tick mark on the x-axis labeled x_2.
6.	In a second trial,	 Add Roman numeral “I” to the “Trial” label previously written.  Write a new label “Trial II”. For each subscript already written, add a comma and the Roman numeral I. So, x_1 becomes $x_{1,\mathrm{I}}$ and x_2 becomes $x_{2,\mathrm{I}}$.
7.	the x-positions of Alice’s starting and ending locations are both increased	 From the tickmark labeled $x_{1,\mathrm{I}}$, draw a curved arrow pointing in the +x direction (in this example, to the right), toward a new tickmark labeled $x_{1,\mathrm{II}}$.  From the second tickmark originally drawn, labeled $x_{2,\mathrm{I}}$, draw another curved arrow pointing in the +x direction (in this example, to the right), toward a new tickmark labeled $x_{2,\mathrm{II}}$.
8.	by the same number of meters.	 Slash each curved arrow with a short segment to indicate congruency.
9.	How does	 Circled question mark indicating start of a shopping list of requested quantities or determinations.
10.	the x-displacement in Alice’s second trial	 x-displacement arrow labeled Δx_{II} drawn from the tickmark labeled $x_{1,\mathrm{II}}$ to the tickmark labeled $x_{2,\mathrm{II}}$.

11.	compare with	 Versus, abbreviated "vs." enclosed in a circle, near the circled question mark.
12.	the x-displacement in her first trial?	 x-displacement arrow, labeled Δx_{I}, connects the tickmark labeled $x_{1,\mathrm{I}}$ to the tickmark labeled $x_{2,\mathrm{I}}$. Each of the x-displacement labels Δx is associated by a caption arrow with the versus symbol.
13.	Choose an option	No illustration needed for high school students. Sixth-grade students might benefit from illustrating the cognitive process of making a choice.
14.	and explain your reasoning.	No illustration needed for high school students. Sixth-grade students might benefit from illustrating the process of writing an explanation.
15.	Choices	Ignore the choices so as to solve the problem as though it were a pure free-response question.

It's perfectly fine if, for example, you used colors or shading to distinguish portions of your illustration corresponding to Trials I and II instead of using caption arrows, as in the example above. That kind of style difference that doesn't change the meaning represented by an illustration is OK.

Additionally, there might occasionally be a case where the key above groups 2 words into a phrase where you grouped 5 words into a phrase, or *vice versa*. This kind of difference in grouping is OK as well, as long as it doesn't cause the corresponding portion of your diagram to have errors or omissions.

However, if there is any error in your illustration or if the steps you took to draw your illustration were substantially inconsistent with the frequently halting phrase-by-phrase process outlined in the table above, you're not ready to proceed. In this case, review the demo video at youtu.be/KMP4T4cyRuc, conceal your original attempt at illustrating practice problem Alice, copy the statement for practice problem Alice to a fresh page of paper, and re-attempt using the reading guide above (or at davidliao.com/read.php) to read practice problem Alice.

4. Day 4: Carefully solving a problem (60-90 minutes)

Get the SiQuENC method (5 minutes)

Print the following problem-solving guide.

Abbreviation	Name	Description
Si	Neatly and graphically represent **Situations**	Create a combination of a **sketch**, a **table**, and/or **labels**. You're done with this step when your work is sufficient for you to understand precisely the scenario in the problem and precisely what you're asked to find, even if the original problem statement is removed. This step should be done once the **reading guide** above is applied to the entire problem statement.
Qu	Graphically represent **Quantities**	Make sure **each relevant physical quantity** has a **corresponding graphical representation**. A graphical representation for a quantity is a portion of a diagram that **could, in principle, be drawn to scale**. A labeled x-displacement arrow is a graphical representation because it could, in principle, be drawn to a correct length if a scale for indicating the length "in real life" that corresponded to 1 cm, say, of length on the page. A lot of this step is often done by the time the reading guide has been applied to the whole problem statement, but some more diagramming might still be possible.
E	Identify allowed starting-point **Equation(s)** and/or inequalities	**E1.** Try to use a note sheet: a. **Pick a note sheet**. Make a **photocopy** of the sheet (by hand is OK). b. In the photocopy, use sections 1.a and 1.b to **check** whether the scenario is **eligible** to be analyzed using the sheet. c. If the photocopy isn't eligible to be applied, send the photocopy back to the pile of notes. **If** the photocopy is **eligible** to be applied, go to the next step.
N	**ANalyze**	**N1.** For the sheet of notes in hand, use section 2.a to **update** your **diagrams**, if possible, and use section 2.b to **organize quantities**. **N2. Substitute** quantities. Perform **qualitative** and **algebraic reasoning**, as much as possible. **N3. Browse** the photocopies you've started filling out. **Check whether** you **can copy** information from one photocopy onto another photocopy. If so, copy the information and **look for additional calculations** that you can now carry out.
C	**Communicate**	**C1.** If you've determined an expression for a requested quantity, **box** the answer (including the **variable**, an **equal sign or inequality sign**, the algebraic or numerical **value**, and, for numerical values, appropriate **units**). **C2.** If asked for an explanation, write an explanation using the **REASoN** method described below.

Use steps E1, N1, N2, C1, and C2 in combination and with repetition, as needed.

Get started with the SiQuENC method (20 minutes)

TIP: Before using a relationship, check eligibility
When you think you want to use a mathematical relationship to solve a problem, check the notes sheet for the relationship to make sure that the problem scenario is eligible for analysis using the relationship.

Read step Si.

Abbreviation	Name	Description
Si	Neatly and graphically represent **Si**tuations	Create a combination of a **sketch**, a **table**, and/or **labels**. You're done with this step when your work is sufficient for you to understand precisely the scenario in the problem and precisely what you're asked to find, even if the original problem statement is removed. This step should be done once the **reading guide** above is applied to the entire problem statement.

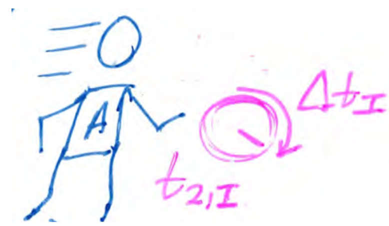
We indeed carefully used the reading guide to read the problem statement, so step Si is done. Read step Qu.

Abbreviation	Name	Description
Qu	Graphically represent **Qu**antities	Make sure **each relevant physical quantity** has a **corresponding graphical representation**. A graphical representation for a quantity is a portion of a diagram that **could, in principle, be drawn to scale**. A labeled x-displacement arrow is a graphical representation because it could, in principle, be drawn to a correct length if a scale for indicating the length "in real life" that corresponded to 1 cm, say, of length on the page. A lot of this step is often done by the time the reading guide has been applied to the whole problem statement, but some more diagramming might still be possible.

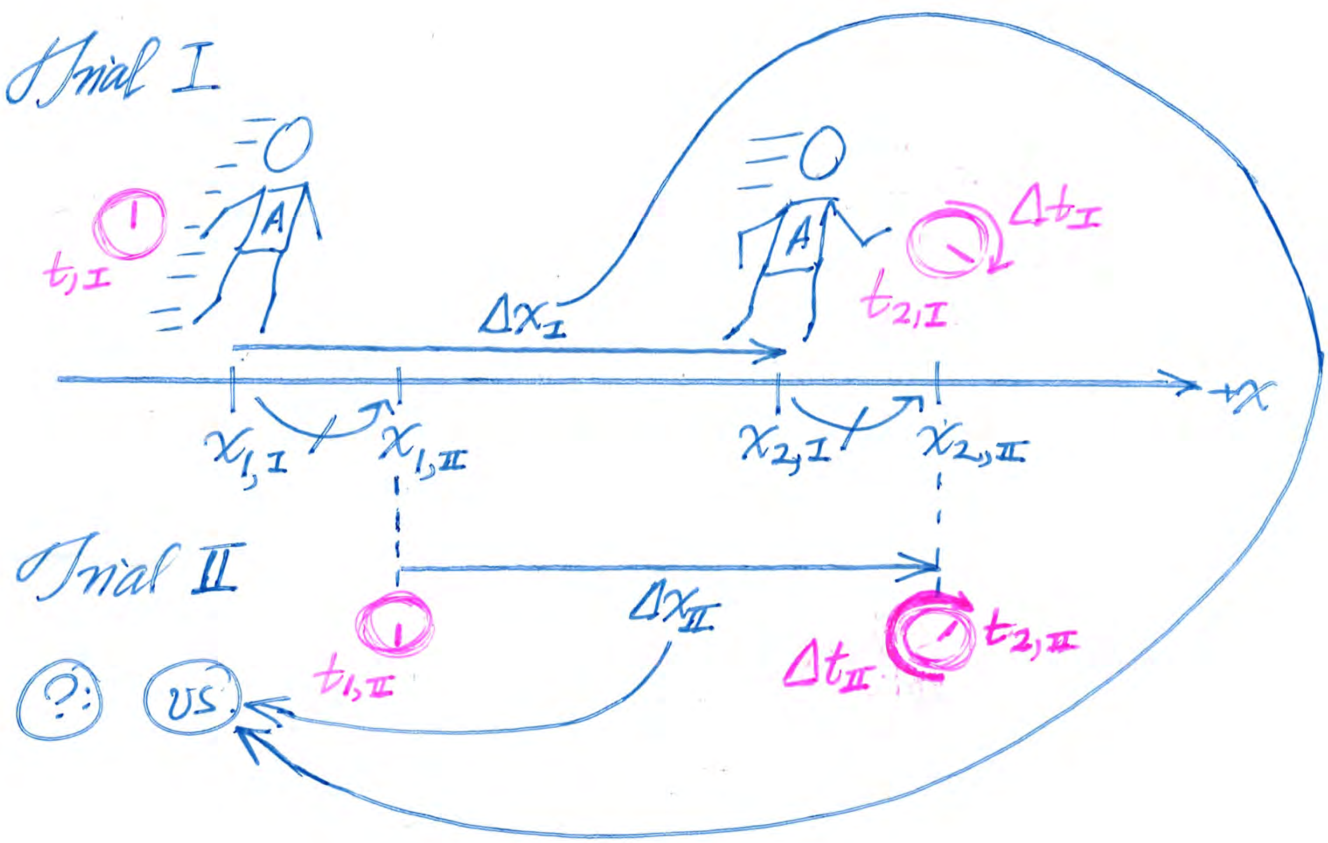
We've *partially* completed step Qu. In our workspace, each x-displacement arrow is graphical representation of a quantity.

We could, however, still add graphical representations of times and elapsed durations by drawing a clock labeled $t_{1,\mathrm{I}}$ near Alice's first snapshot,

a clock showing a later time labeled $t_{2,\mathrm{I}}$ near Alice's second snapshot,



an elapsed duration arc labeled Δt_{I} showing the amount of time that elapsed between the times labeled $t_{1,\text{I}}$ and $t_{2,\text{I}}$ and similar markings for Trial 2. The problem statement doesn't give information on how the time values or elapsed durations in the two experimental trials compare, so we won't particularly worry about drawing elapsed durations to scale.



(It turns out the time values are not that interesting when solving this problem, but when you're starting out in physics, it's a good exercise to practice graphically representing as many quantities as you can).

Go to step E, "Identify allowed starting-point **E̲quation(s)** and/or inequalities". Read step E1 through substep a.

> **E1.** Try to use a note sheet:
> a. **Pick a note sheet**. Make a **photocopy** of the sheet (by hand is OK).

In the future, we'll have lots of note sheets to sift through, but for now, we'll pick the lone note sheet we've created and make a photocopy below.

Elapsed duration and x-displacement

1. Is scenario eligible for analysis with this sheet?

1.a. Inspect sketch

1.b. Document qualifications

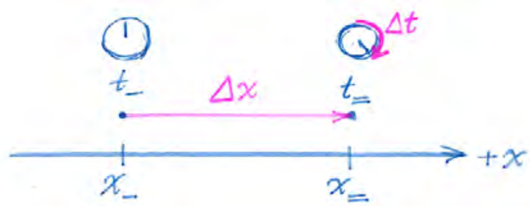
System (in bubble):	
+x direction (draw arrow):	
Label for _st snapshot:	
Label for _nd snapshot:	

2. If scenario is eligible, continue using this sheet.

2.a. Draw a diagram

Elapsed duration curve – directed arc traces sweep of clock hand tip

Displacement vector – arrow from _st position dot to _nd position dot



2.b. Fill in and compute values in a form

_st time	+	Elapsed duration	=	_nd time
$t_{\_}$	+	Δt	=	$t_{\_}$
	+		=	

_st x-position	+	x-displacement	=	_nd x-position
$x_{\_}$	+	Δx	=	$x_{\_}$
	+		=	

Read substep E1.b.

b. In the photocopy, use sections 1.a and 1.b to **check** whether the scenario is **eligible** to be analyzed using the sheet.

Look at panel 1.a from the note sheet.

1.a. Inspect sketch

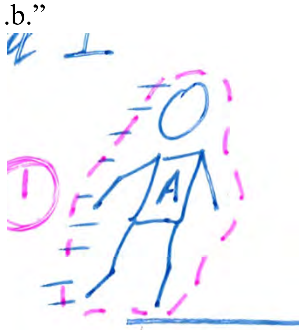
Consider whether there's anything in the diagram that obviously doesn't match the illustration we've made for the practice problem. Even though the note sheet shows two snapshots of a moving cart and the illustration for our practice problem shows snapshots for a moving person, we seem to be visualizing related situations. It remains plausible that the note sheet we've chosen can be used to analyze the scenario in our practice problem.

Look at panel 1.b from the note sheet. Try to fill in this entire panel. If there's an entry in this panel that can be filled in only by contradicting the practice problem statement, the note sheet we've picked can't be applied to the current problem.

1.b. Document qualifications

System (in bubble):	
+x direction (draw arrow):	
Label for _st snapshot:	
Label for _nd snapshot:	

Read "System (in bubble):". Ask, "What can play the role of the System?" Realize, "Oops, I forgot to draw a dashed bubble to indicate a system. I want to focus on Alice, so I'll draw a dashed bubble around Alice and write 'Alice' in the 'System' entry in notes panel 1.b."



Read "+x direction (draw arrow):" Ask, "What can play the role of the +x direction?" Notice, "I've drawn an x-axis pointed toward the right, so the +x direction I'm using is rightward, and I'll draw a rightward arrow for my +x direction in notes panel 1.b."

Read "Label for _st snapshot:". Ask, "What can play the role of the _st snapshot?" Notice, "I've drawn four snapshots (two in Trial 1 and two in Trial 2). Any of these could be a _st snapshot. Let's focus on Trial 1's earliest snapshot. So, I'll let snapshot 1,I be my _st snapshot and fill in the corresponding entry in notes panel 1.b."

Read "Label for _nd snapshot:". Ask, "What can play the role of the _nd snapshot?" Notice, "All four snapshots illustrated are snapshots for the same system, Alice, so any of these four snapshots can play the role of the _nd snapshot. Let's try, for example letting snapshot 2,I play the role of the _nd snapshot and fill in the corresponding entry in notes panel 1.b. accordingly."

At this point, notes panel 1.b should look like the following.

1.b. Document qualifications

System (in bubble):	Alice
+x direction (draw arrow):	→
Label for _st snapshot:	1,I
Label for _nd snapshot:	2,I

Because we were able to look at panel 1.a without noticing an obvious inconsistency between the sketch there and our illustration for our practice problem and because we filled out the entries in panel 1.b without running into any contradictions, our practice problem is eligible to be analyzed using the note sheet we picked.

Read substep E1.c.

c. If the photocopy isn't eligible to be applied, send the photocopy back to the pile of notes. **If** the photocopy is **eligible** to be applied, go to the next step.

Go to step N, "**A<u>N</u>alyze**". Read substep N1.

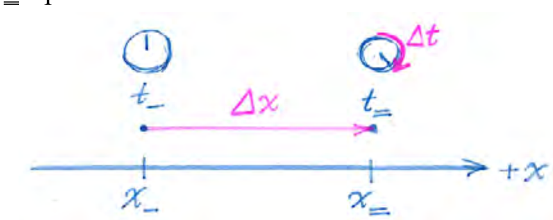
N1. For the sheet of notes in hand, use section 2.a to **update** your **diagrams**, if possible, and use section 2.b to **organize quantities**.

In the notes sheet, look at panel 2.a.

<u>2.a. Draw a diagram</u>

Elapsed duration curve – directed arc traces sweep of clock hand tip

Displacement vector – arrow from _st position dot to ‗nd position dot



This panel reminds us to draw an elapsed duration curve for the time interval from snapshot 1,I to snapshot 2,I and a displacement vector connecting the x-position in snapshot 1,I to the x-position in snapshot 2,I, but we've already done both these things. So, let's move on to panel 2.b.

<u>2.b. Fill in and compute values in a form</u>

_st time	+	Elapsed duration	=	‗nd time
$t_{\_}$	+	Δt	=	$t_{=}$
	+		=	
_st x-position	+	x-displacement	=	‗nd x-position
$x_{\_}$	+	Δx	=	$x_{=}$
	+		=	

In panel 1.b, we indicated that the _st snapshot was 1,I and the ‗nd snapshot was 2,I. We update the labeled subscripts in panel 2.b accordingly.

2.b. Fill in and compute values in a form

1,Ist time	+	Elapsed duration	=	2,Ind time
$t_{1,\mathrm{I}}$	+	Δt	=	$t_{2,\mathrm{I}}$
	+		=	
1,Ist x-position	+	x-displacement	=	2,Ind x-position
$x_{1,\mathrm{I}}$	+	Δx	=	$x_{2,\mathrm{I}}$
	+		=	

Now, we fill in as many of the blank boxes in panel 2.b as we can. We can use numerical and algebraic expressions, as available, from the illustration we created when we read the problem statement. With the sensible subscripts we chose when we labeled quantities in the illustration we created for our practice problem, most of the algebraic symbols we write into the boxes match the labels above the boxes.

2.b. Fill in and compute values in a form

1,Ist time	+	Elapsed duration	=	2,Ind time
$t_{1,\mathrm{I}}$	+	Δt	=	$t_{2,\mathrm{I}}$
$t_{1,\mathrm{I}}$	+	Δt_{I}	=	$t_{2,\mathrm{I}}$
1,Ist x-position	+	x-displacement	=	2,Ind x-position
$x_{1,\mathrm{I}}$	+	Δx	=	$x_{2,\mathrm{I}}$
$x_{1,\mathrm{I}}$	+	Δx_{I}	=	$x_{2,\mathrm{I}}$

The only difference is that the elapsed duration and x-displacement we write into the form boxes each have a subscript Roman numeral I. The photocopy of the notes sheet we're using is as filled-in as possible.

Read substep N2.

> **N2. Substitute** quantities. Perform **qualitative** and **algebraic reasoning**, as much as possible.

There were no numerical values provided, and we're not able to carry out any arithmetic calculations to obtain a numerical expression for anything.

Read substep N3.

> **N3. Browse** the photocopies you've started filling out. **Check whether** you **can copy** information from one photocopy onto another photocopy. If so, copy the information and **look for additional calculations** that you can now carry out.

We haven't started filling out any other photocopies of notes sheets, so there are no other photocopies for us to copy information to from the lone photocopy we have in hand.

Read step C.

C	**C̲ommunicate**	**C1.** If you've determined an expression for a requested quantity, **box** the answer (including the **variable**, an **equal sign or inequality sign**, the algebraic or numerical **value**, and, for numerical values, appropriate **units**). **C2.** If asked for an explanation, write an explanation using the **REASoN** method described below.

At this moment, we haven't determined the value of a quantity (and the problem wasn't asking for the value of quantity, anyway). We've also not written out enough reasoning to carry out the requested comparison of the x-displacements from the two experimental trials. So, neither substep C1 nor substep C2 can be performed.

In this case, it happens to help to return to step E. Substeps E1, N1, and N2 take us through a vetting and analysis process similar to the one we just used leading us to fill out a second photocopy of our notes sheet for Elapsed duration and x-displacement, but this time for Trial 2. In particular, panel 1.b will look like this.

1.b. Document qualifications

System (in bubble):	Alice
+x direction (draw arrow):	→
Label for _st snapshot:	1,II
Label for _nd snapshot:	2,II

And, panel 2.b will look like this.

2.b. Fill in and compute values in a form

1,IIst time	+	Elapsed duration	=	2,IInd time
$t_{1,\mathrm{II}}$	+	Δt	=	$t_{2,\mathrm{II}}$
$t_{1,\mathrm{II}}$	+	Δt_{II}	=	$t_{2,\mathrm{II}}$
1,IIst x-position	+	x-displacement	=	2,IInd x-position
$x_{1,\mathrm{II}}$	+	Δx	=	$x_{2,\mathrm{II}}$
$x_{1,\mathrm{II}}$	+	Δx_{II}	=	$x_{2,\mathrm{II}}$

If you're really working on a problem independently (with no help), you might spend 20 minutes at this point reading out descriptions of substeps E1, N1, N2, N3, C1, and C2 and asking yourself, after reading each, whether the substep can be used. Eventually, you might realize that you want to read substep N2 again.

N2. Substitute quantities. Perform **qualitative** and **algebraic reasoning**, as much as possible.

In particular, quantities to be substituted can be identified by browsing the illustration for the practice problem. We had drawn congruent marks to represent that $x_{1,\mathrm{II}}$ exceeded $x_{1,\mathrm{I}}$ by an amount equal to the amount by which $x_{2,\mathrm{II}}$ exceeded $x_{1,\mathrm{II}}$.

If we name this amount of exceedance, say, L, we can say that $x_{1,\text{II}}$ is L more than $x_{1,\text{I}}$ and that $x_{2,\text{II}}$ is L more than $x_{2,\text{I}}$. In equations, we write $x_{1,\text{I}} + L = x_{1,\text{II}}$ and $x_{2,\text{I}} + L = x_{2,\text{II}}$. This lets us update panel 2.b in our notes sheet photocopy for Trial 2 by replacing the x-position entries.

2.b. Fill in and compute values in a form

1,II[st] time	+	Elapsed duration	=	2,II[nd] time
$t_{1,\text{II}}$	+	Δt	=	$t_{2,\text{II}}$
$t_{1,\text{II}}$	+	Δt_{II}	=	$t_{2,\text{II}}$
1,II[st] x-position	+	x-displacement	=	2,II[nd] x-position
$x_{1,\text{II}}$	+	Δx	=	$x_{2,\text{II}}$
$x_{1,\text{I}} + L$	+	Δx_{II}	=	$x_{2,\text{I}} + L$

But, then this immediately lets us subtract L from both sides of the equation containing Δx_{II}:

$$x_{1,\text{I}} + \cancel{L} + \Delta x_{\text{II}} = x_{2,\text{I}} + \cancel{L}$$

$$x_{1,\text{I}} + \Delta x_{\text{II}} = x_{2,\text{I}} \qquad (2)$$

Compare equation (2) with the equation at the bottom of panel 2.b for the photocopy of the notes sheet we used to analyze Trial 1:

$$x_{1,\text{I}} + \Delta x_{\text{I}} = x_{2,\text{I}} \qquad (1)$$

Equations (1) and (2) are identical except that the x-displacement in equation (2) is from Trial 2 and the x-displacement in equation (1) is from Trial 1. The two x-displacements are found in otherwise the same equation and, so, constrained to have identical value.

$$\Delta x_{\text{I}} = \Delta x_{\text{II}}$$

Note: It's possible for two quantities found in otherwise identical equations to have different values. This occurs when the solution to an equation is not unique. However, students in Honors Physics and AP Physics 1 are not expected to demonstrate that, for example, there is only one value of Δx_{I} consistent with equation (1).

We can now mark the correct choice from the problem statement.

B. Alice's 2[nd] x-displacement equals her 1[st] x-displacement.

We must still write an explanation.

Get the REASoN method for writing explanations (5 minutes)

Print out the following scaffold.

Abbreviation	Step	Instruction
R	**<u>R</u>elationship**/Rule	State a starting-point equation from allowed knowledge (as it appears in panel 2.b on an unused sheet of notes).
E	**<u>E</u>qual**/Same/Unchanged	State what quantities in the main relationship remain the same between two trials. Or, state what quantities are substituted in for variables in the main relationship.
A	**<u>A</u>ltered**/Different	State what quantities change (and how) in the main relationship between two trials.
So	**<u>So</u> what?**	Draw a mathematical conclusion.
N	**<u>N</u>ext?**	Is there anything else to determine next? Have all requests in the problem statement been satisfied? Do you need to carry out some more qualitative or algebraic reasoning? Do you need to start a new cycle of REASoNing that states a physics relationship?

Use the REASoN method to write an explanation (30 minutes)

Read step R.

Abbreviation	Step	Instruction
R	**<u>R</u>elationship**/Rule	State a starting-point equation from allowed knowledge (as it appears in panel 2.b on an unused sheet of notes).

When an experiment consists of multiple trials that are prepared in similar ways, it's often more convenient to write about common features of both trials, rather than talking about how an equation could be applied specifically to one trial (and then having to talk later about how the same equation could be separately applied to another trial). For example, the main definition enabling our reasoning is the definition of x-displacement. Try writing something like this.

"By the definition of x-displacement, Alice's x-displacement in a trial is the amount of change in x-position that needs to be added to Alice's initial x-position in that trial to obtain Alice's final x-position in that trial."

TIP: Show the grader which physics note sheets you use
When asked to "explain your reasoning", indicate the relevant portions of the titles of the note sheets you use and state the meanings of the corresponding mathematical relationships. You can sometimes omit this step if the instruction asks you to "briefly explain", rather than to "explain your reasoning".

Read step E.

Abbreviation	Step	Instruction
E	**<u>E</u>qual**/Same/Unchanged	State what quantities in the main relationship remain the same between two trials. Or, state what quantities are substituted in for variables in the main relationship.

The problem statement doesn't give any quantities that remain the same in the two trials. Sometimes there's nothing to write for a step, and that's OK.

The amount by which $x_{1,\mathrm{II}}$ exceeds $x_{1,\mathrm{I}}$ indeed equals the amount by which $x_{2,\mathrm{II}}$ exceeds $x_{2,\mathrm{I}}$, as mentioned before. It doesn't matter much, but I personally think of this insight as belonging more to the next step, step A.

Abbreviation	Step	Instruction
A	**Altered**/Different	State what quantities change (and how) in the main relationship between two trials.

"The amount by which the initial x-position is increased equals the amount by which the final x-position is increased."

Read step So.

Abbreviation	Step	Instruction
So	**So what?**	Draw a mathematical conclusion.

"So, the difference (x-displacement) between the initial and final x-positions must be the same in the two trials."

TIP: Algebra-based physics courses don't require derivations of algebraic properties
Honors Physics and AP Physics 1 are "algebra-based" physics courses. Ability to perform algebra is presumed (unless obviously rebutted by incorrect work) to be proficient. So, you don't really have to say, "The difference between two quantities remains unchanged when the quantities themselves change by the same amount."

TIP: Enclosing an expression in parentheses can communicate as much as a sentence
Consider replacing a pair of sentences like "Sally walked to a final x-position. Her final x-position was 5 m." with a single sentence like "Sally walked to a final x-position (5 m)."

It's OK to "sneak in" the x-displacement, enclosed in parentheses, next to its synonym, "difference" (between the initial and final x-positions). You previously cited the definition of x-displacement, so the grader can already tell that you have the physics knowledge needed to know that the x-displacement and the difference between initial and final x-positions are the same.

Read step N.

Abbreviation	Step	Instruction
N	**Next?**	Is there anything else to determine next? Have all requests in the problem statement been satisfied? Do you need to carry out some more qualitative or algebraic reasoning? Do you need to start a new cycle of REASoNing that states a physics relationship?

We've already answered the question, so no additional work is needed.

The completed argument looks like the following.

REASoN step	Sentence(s)
Relationship/rule	By the definition of x-displacement, Alice's x-displacement in a trial is the amount of change in x-position that needs to be added to Alice's initial x-position in that trial to obtain Alice's final x-position in that trial.
Equal/unchanged	
Altered/different	The amount by which the initial x-position is increased equals the amount by which the final x-position is increased.
So what?	So the difference (x-displacement) between the initial and final x-positions must be the same in the two trials.
Next?	(No)

TIP: Strive to avoid including the third-person pronoun "it" in explanations
To avoid ambiguity, repeat a noun instead of using the third-person pronoun "it".

In casual speech, someone might conclude the argument above by saying, "So, it must be the same in the two trials." I award zero points to this sentence because the antecedent to the pronoun "it" could be any of the nouns previously mentioned in the written explanation. When grading, I don't let myself assume the student knows enough physics to intend the correct antecedent ("x-displacement").

5. Day 5: Independent practice (2 hours)

Attempt the following problem.

Practice Problem: Bob. In an experimental trial, Bob walks along the x-axis from one location to another. In a second trial, the x-positions of Bob's starting and ending locations are both doubled. How does the x-displacement in Bob's second trial compare with the x-displacement in his first trial? Choose an option and explain your reasoning.

A. Bob's 2nd x-displacement is greater than his 1st x-displacement.
B. Bob's 2nd x-displacement equals his 1st x-displacement.
C. Bob's 2nd x-displacement is less than his 1st x-displacement.
D. Cannot be determined.

Key

Check your answer against the following key.

Answer choice: (D) Cannot be determined.

Explanation:

REASoN step	Example
Relationship/rule	The x-displacement is defined according to $x_1 + \Delta x = x_2$.
Equal/unchanged	
Altered/different	The initial and final x-positions double.
So what?	So, the x-displacement doubles.
Next?	(Yes, there remains more to analyze next, but not through the use of additional physics equations. The remaining reasoning is carried out through mathematical reasoning alone). Doubling a positive x-displacement would mean an increase in x-displacement. Doubling an x-displacement of zero would mean no change in x-displacement. Doubling a negative x-displacement would mean a decrease in x-displacement. Nothing in the problem statement lets us determine whether the x-displacement in the first trial is positive, zero, or negative. So, we have no way to know whether the x-displacement increased, stayed the same, or decreased.

6. Exit interview (10 minutes)

How did it go? Did you attempt problem "Bob" with **exactly** the same detail and tedious care as demonstrated for problem **Alice**?

As you plan your time (including choosing your course schedule, if you're reading this before the course-drop deadline), consider the following.

Some things get harder

- The equations we've covered in this bootcamp are more-or-less the two simplest equations (likely among the first equations to be learned) in CP Physics, Enriched Physics, Honors Physics, and AP Physics 1.
- Newton's 2nd Law tests around the middle of the fall semester (near the end of the 1st marking period) are brutal. Tutors should receive a burst of phone calls and text messages around this time from families in which previously straight-A students are crying about getting 25%-40% on exams. Imagine carrying out logical reasoning based on allowed knowledge, as we have demonstrated in this packet, *but* with the additional challenge that many people have deeply internal instincts and feelings that make it very difficult to work with the lynchpin unit of allowed knowledge (Newton's 2nd Law) that is the foundation for 80% of the reasoning for a test.
- Problem-solving gets more difficult when multiple equations have been learned and you have to carefully sift through them to find the ones most relevant to a problem.
- The trick of giving you a practice problem and a demonstration problem that were superficially similar and just slightly different so that the tiniest ounce of carelessness is enough to cause catastrophic failure is routine in AP Physics 1. In both Honors Physics and AP Physics 1, reading is tested severely by writing problem statements that are easily misread.
- Mercy points are not really given out for broken written explanations. If you don't have time to learn to write written explanations, enroll in CP Physics or Enriched Physics, not Honors Physics or AP Physics 1.
- You should use the extremely tedious methods in this bootcamp packet until you can carry them out reliably without assistance. At that point, you can start to relax and accelerate your technique (do this in small steps, paying attention to whether you've lost substantial accuracy and need to slow down).

Some things get better

- Multiple chapters of knowledge can be organized into production schemas (example: https://davidliao.com/handouts/Physics/11%20Make%20and%20solve%20physics%20problems/T.0.2%20Production%20system%20algebra-based%20physics.pdf).
- If you get used to the kind of notetaking, reading, reasoning, and writing in this packet, just reading a book and carefully taking notes, without doing practice problems, might be enough to prepare you to solve unfamiliar problems (given unlimited time). Practice is still good for becoming fast enough for timed tests.

7. References

Obtain knowledge through scientific experimentation

- E. Etkina and A. Van Heuvelen, "Investigative Science Learning Environment – A Science Process Approach to Learning Physics", in *Research Based Reform of University Physics*, E.F. Redish and P. Cooney (eds)., online: http://per-central.org/per_reviews/media/volume1/ISLE-2007.pdf.
- D. Liao and T.D. Tlsty, "Evolutionary game theory for physical and biological scientists. I. Training and validating population dynamics equations", *Interface Focus*, **4**, 20140037 (2014) doi:10.1098/rsfs.2014.0037.

Lay out your notes so other people know that you're using multiple representations

- R.D. Knight, B. Jones, and S. Field, *College Physics: A Strategic Approach* 3rd ed. Pearson, (2014), amazon: 0321879724.
- E. Etkina, M. Gentile, and A. Van Heuvelen, *College Physics* (Pearson, Glenview, IL, 2014), amazon:0321715357.
- H.D. Young and R.A. Freedman, *University Physics with Modern Physics* 14th ed. Pearson, (2015), amazon: 0321973615.

Use multiple representations when solving problems

- J. Larkin, J. McDermott, D. Simon, and H. Simon, "Expert and novice performance in solving physics problems", *Sci.* **208**, 1335-1342 (1980) doi:10.1126/science.208.4450.1335.
- D. Liao, "A SiQuENC for solving physics problems", *Phys. Teach.*, **56**, 264-265 (2018) doi:10.1119/1.5028250.

Deliberate thinking can be used to reject invalid reasoning

- S. Ismael and M. Krujevskaia, "Toward helping students develop error detection skills", *Physics Education Research Conference 2023*, Sacramento, CA, (July 19-20, 2023) doi.org/10.1119/perc.2023.pr.Ismael.

Visually represent the incorporation of a selected concept into a line of reasoning

- J. Speirs, W. Ferm, Jr., M. Stetzer, and B. Lindsey, "Probing student ability to construct reasoning chains: a new methodology." Paper presented at the *Physics Education Research Conference 2016*, Sacramento, CA, (July 20-21, 2016). doi:10.1119/perc.2016.pr.077.

Write explanations that include necessary components of arguments

- S.E. Toulmin, *The Uses of Argument*, Cambridge University Press (1958).
- K.L. McNeill, D.J. Lizotte, J. Krajcik, and R.W. Marx, "Supporting students' construction of scientific explanations by fading scaffolds in instructional materials", *J. Learn. Sci.*, **15**, 153-191 (2006). doi:10.1207/s15327809jls1502_1.
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