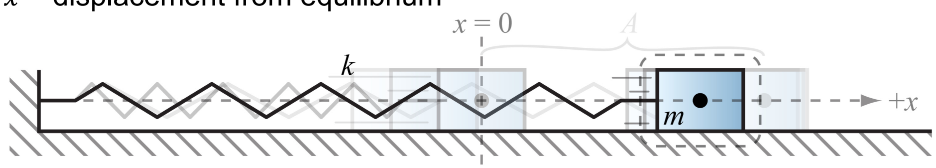
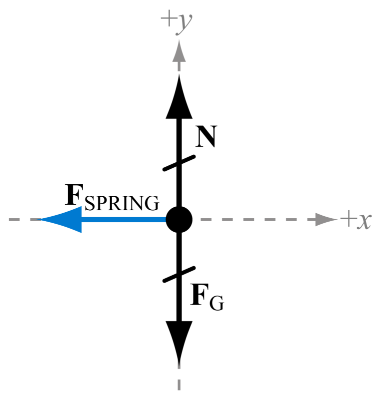
Simple harmonic motion (SHM) is the one-dimensional shadow of UCM

Mass on a spring

x – displacement from equilibrium





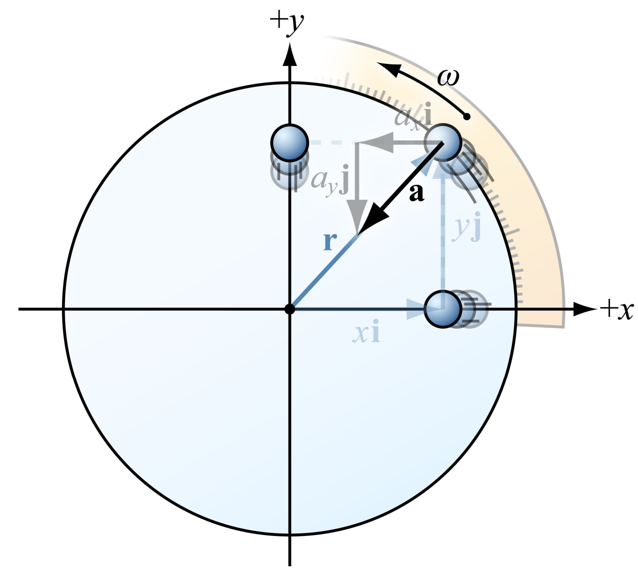
Spring force
Direction: Restoring
Magnitude: ∝ Displacement

$$a_x = \frac{\Sigma F_x}{m}$$

$$a_x = \frac{-k|\Delta x|}{m}$$

$$a_x = \frac{-kx}{m}$$

$$a_x = -\frac{k}{m}x$$



$$\left|\frac{a_x}{a_{\text{IN}}}\right| = \left|\frac{x}{r}\right|$$

$$|a_x| = \left|\frac{a_{\text{IN}}}{r}\right||x|$$

$$a_{\text{IN}} = \frac{v^2}{r} = \omega^2 r$$

$$\frac{a_{\text{IN}}}{r} = \omega^2$$

$$|a_x| = \omega^2|x|$$

$$a_x = -\omega^2 x$$

ω – angular velocity of UCM that completes one lap in the same duration of time that SHM of interest completes one cycle of oscillation

A – amplitude (maximum linear or angular distance from equilibrium)

T – period (repetition time)

When magnitude of restoring net force (torque) is ∝ (angular) displacement, T is independent of A

f – frequency (oscillations per unit time)

$$\omega^2 = \frac{k}{m}$$

$$x(t) = A\cos\left(\sqrt{\frac{k}{m}}t + \delta\right) = A\cos\left(\frac{2\pi}{T}t + \delta\right)$$

$$\omega = \frac{2\pi}{T} \Rightarrow T = \frac{2\pi}{\omega}$$

$$T = \frac{1}{f} = 2\pi\sqrt{\frac{m}{k}}$$