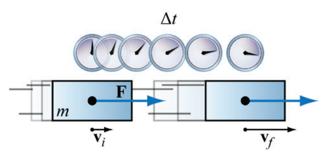
Newton’s second law can be re-expressed

$$\vec{\mathbf{a}} = \frac{\sum \vec{\mathbf{F}}}{m}$$

Impulse-momentum theorem (focus on force and time)



Algebra-based physics

Consider time-averaged force

$$\vec{\mathbf{a}}_{\text{AVG}} = \frac{\sum \vec{\mathbf{F}}_{\text{AVG}}}{m}$$

$$\frac{\sum \vec{\mathbf{F}}_{\text{AVG}}}{m} = \frac{\Delta \vec{\mathbf{v}}}{\Delta t}$$

$$\sum \vec{\mathbf{F}}_{\text{AVG}}\, \Delta t = m\Delta \vec{\mathbf{v}}$$

$$= m(\vec{\mathbf{v}}_f - \vec{\mathbf{v}}_i)$$

$$= m\vec{\mathbf{v}}_f - m\vec{\mathbf{v}}_i$$

$$\underbrace{\sum \vec{\mathbf{F}}_{\text{AVG}}\, \Delta t}_{\Delta \vec{\mathbf{J}}_{\Sigma \vec{\mathbf{F}}}} = \Delta\left(\underbrace{m\vec{\mathbf{v}}}_{\vec{\mathbf{p}}}\right)$$

Calculus-based physics

$$\frac{\sum \vec{\mathbf{F}}}{m} = \frac{\mathrm{d}\vec{\mathbf{v}}}{\mathrm{d}t}$$

$$\int_{t=t_i}^{t=t_f} \Sigma \vec{\mathbf{F}}\, \mathrm{d}t = \int_{\vec{\mathbf{v}}=\vec{\mathbf{v}}_i}^{\vec{\mathbf{v}}=\vec{\mathbf{v}}_f} m\, \mathrm{d}\vec{\mathbf{v}}$$

$$= m \int_{\vec{\mathbf{v}}=\vec{\mathbf{v}}_i}^{\vec{\mathbf{v}}=\vec{\mathbf{v}}_f} \mathrm{d}\vec{\mathbf{v}}$$

$$= m(\vec{\mathbf{v}}_f - \vec{\mathbf{v}}_i)$$

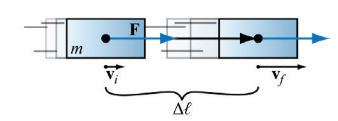
$$= m\vec{\mathbf{v}}_f - m\vec{\mathbf{v}}_i$$

$$\underbrace{\int_{t=t_i}^{t=t_f} \Sigma \vec{\mathbf{F}}\, \mathrm{d}t}_{\Delta \vec{\mathbf{J}}_{\Sigma \vec{\mathbf{F}}}} = \Delta\left(\underbrace{m\vec{\mathbf{v}}}_{\vec{\mathbf{p}}}\right)$$

$$\Delta \vec{\mathbf{J}}_{\Sigma \vec{\mathbf{F}}} = \Delta \vec{\mathbf{p}}$$

$$\vec{\mathbf{p}}_i + \overbrace{\sum_{\text{F}} \Delta \vec{\mathbf{J}}_{\text{F}}}^{\Delta \vec{\mathbf{J}}_{\Sigma \vec{\mathbf{F}}}} = \vec{\mathbf{p}}_f$$

Work-energy theorem (focus on force and position)



Algebra-based physics

Consider uniform acceleration in 1-d

$$v_i^2 + 2a\Delta\ell = v_f^2$$

$$v_i^2 + 2\left(\frac{\sum F}{m}\right)\Delta\ell = v_f^2$$

$$\underbrace{\frac{1}{2}mv_i^2}_{K_i} + \underbrace{\left(\sum F\right)\Delta\ell}_{\Delta W_{\Sigma \vec{\mathbf{F}}}} = \underbrace{\frac{1}{2}mv_f^2}_{K_f}$$

Calculus-based physics

$$\frac{\mathrm{d}\vec{\mathbf{v}}}{\mathrm{d}t} = \frac{\sum \vec{\mathbf{F}}}{m}$$

$$\int_{\vec{\mathbf{r}}=\vec{\mathbf{r}}_i}^{\vec{\mathbf{r}}=\vec{\mathbf{r}}_f} \Sigma \vec{\mathbf{F}} \cdot \mathrm{d}\vec{\boldsymbol{\ell}} = \int_{\vec{\mathbf{r}}=\vec{\mathbf{r}}_i}^{\vec{\mathbf{r}}=\vec{\mathbf{r}}_f} m \frac{\mathrm{d}\vec{\mathbf{v}}}{\mathrm{d}t} \cdot \mathrm{d}\vec{\boldsymbol{\ell}}$$

$$= \int_{\vec{\mathbf{v}}=\vec{\mathbf{v}}_i}^{\vec{\mathbf{v}}=\vec{\mathbf{v}}_f} m\, \mathrm{d}\vec{\mathbf{v}} \cdot \frac{\mathrm{d}\vec{\boldsymbol{\ell}}}{\mathrm{d}t}$$

$$= \int_{\vec{\mathbf{v}}=\vec{\mathbf{v}}_i}^{\vec{\mathbf{v}}=\vec{\mathbf{v}}_f} m\, \mathrm{d}\vec{\mathbf{v}} \cdot \vec{\mathbf{v}}$$

$$= \int_{\vec{\mathbf{v}}=\vec{\mathbf{v}}_i}^{\vec{\mathbf{v}}=\vec{\mathbf{v}}_f} m \frac{\mathrm{d}(\vec{\mathbf{v}} \cdot \vec{\mathbf{v}})}{2}$$

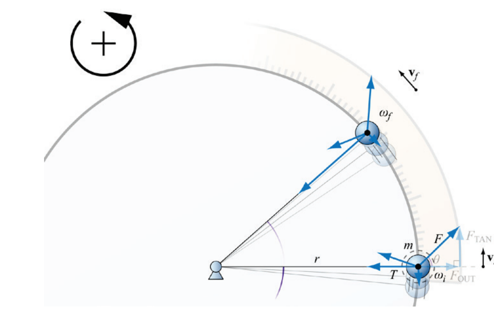
$$= \frac{1}{2}m \int_{v=v_i}^{v=v_f} \mathrm{d}(v^2)$$

$$\underbrace{\int_{\vec{\mathbf{r}}=\vec{\mathbf{r}}_i}^{\vec{\mathbf{r}}=\vec{\mathbf{r}}_f} \Sigma \vec{\mathbf{F}} \cdot \mathrm{d}\vec{\boldsymbol{\ell}}}_{\Delta W_{\Sigma \vec{\mathbf{F}}}} = \underbrace{\frac{1}{2}mv_f^2}_{K_f} - \underbrace{\frac{1}{2}mv_i^2}_{K_i}$$

$$\Delta W_{\Sigma \vec{\mathbf{F}}} = \Delta K$$

$$K_i + \Delta W_{\Sigma \vec{\mathbf{F}}} = K_f$$

Rotational formulation for Newton’s second law (change coordinate system)



$$a_{\text{IN}}(-\hat{\mathbf{r}}) + a_{\text{TAN}}\hat{\mathbf{t}} = \frac{\sum F_{\text{IN}}(-\hat{\mathbf{r}}) + \sum F_{\text{TAN}}\hat{\mathbf{t}}}{m}$$

$$a_{\text{IN}}(-\hat{\mathbf{r}}) + r\alpha\hat{\mathbf{t}} = \frac{\sum F_{\text{IN}}(-\hat{\mathbf{r}}) + \sum F_{\text{TAN}}\hat{\mathbf{t}}}{m}$$

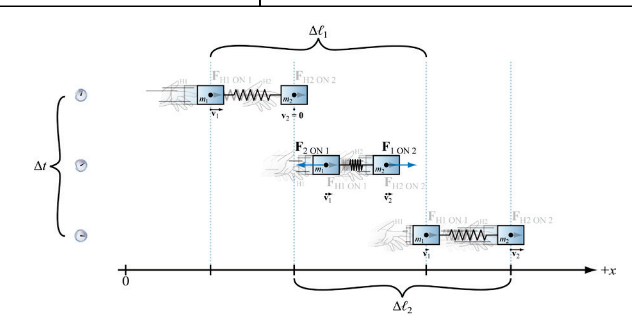
$$r\alpha = \frac{\sum F_{\text{TAN}}}{m}$$

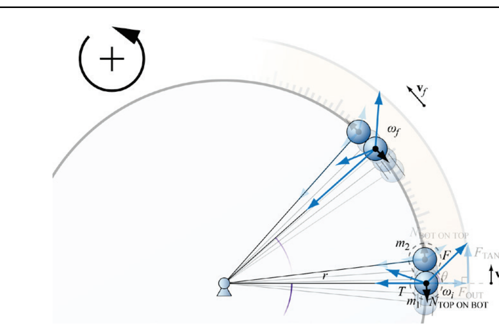
$$\alpha = \frac{\sum F_{\text{TAN}}}{mr}$$

$$\alpha = \frac{\sum rF_{\text{TAN}}}{mr^2}$$

$$\alpha = \frac{\overbrace{\sum rF \sin\theta}^{\tau}}{\underbrace{mr^2}_{I}}$$

$$\alpha = \frac{\sum \tau}{I}$$





For system of N particles

$$\vec{\mathbf{p}}_{1,i} + \Delta \vec{\mathbf{J}}_{\Sigma \vec{\mathbf{F}}\ \text{ON 1}} = \vec{\mathbf{p}}_{1,f}$$
$$\vec{\mathbf{p}}_{2,i} + \Delta \vec{\mathbf{J}}_{\Sigma \vec{\mathbf{F}}\ \text{ON 2}} = \vec{\mathbf{p}}_{2,f}$$
$$\vdots$$
$$\vec{\mathbf{p}}_{N,i} + \Delta \vec{\mathbf{J}}_{\Sigma \vec{\mathbf{F}}\ \text{ON}\ N} = \vec{\mathbf{p}}_{N,f}$$

$$\Sigma \vec{\mathbf{p}}_i + \Sigma \Delta \vec{\mathbf{J}}_{\Sigma \vec{\mathbf{F}}} = \Sigma \vec{\mathbf{p}}_f$$

$$\Sigma \vec{\mathbf{p}}_i + \sum_{\substack{\text{EXT}\\ \text{ON SYS}}} \Delta \vec{\mathbf{J}} + \sum_{\text{INT}} \Delta \vec{\mathbf{J}} = \Sigma \vec{\mathbf{p}}_f$$

Internal Newton’s third law interaction force pairs have opposite directions. The magnitude vs. time plots for each force from a third law pair is identical. Thus, pairs of impulses corresponding to third law force pairs cancel out.

$$\Sigma \vec{\mathbf{p}}_i + \overbrace{\sum_{\substack{\text{EXT}\\ \text{ON SYS}}} \Delta \vec{\mathbf{J}}_{\text{F}}}^{\Delta \vec{\mathbf{J}}_{\Sigma \vec{\mathbf{F}},\text{EXT ON SYS}}} = \Sigma \vec{\mathbf{p}}_f$$

For system of N particles

$$K_{1,i} + \Delta W_{\Sigma \vec{\mathbf{F}}\ \text{ON 1}} = K_{1,f}$$
$$K_{2,i} + \Delta W_{\Sigma \vec{\mathbf{F}}\ \text{ON 2}} = K_{2,f}$$
$$\vdots$$
$$K_{N,i} + \Delta W_{\Sigma \vec{\mathbf{F}}\ \text{ON}\ N} = K_{N,f}$$

$$\Sigma K_i + \Sigma \Delta W_{\Sigma \vec{\mathbf{F}}} = \Sigma K_f$$

$$\Sigma K_i + \sum_{\substack{\text{EXT}\\ \text{ON SYS}}} \Delta W + \sum_{\text{INT}} \Delta W = \Sigma K_f$$

When the relative spatial arrangement of a collection of objects is changed, the total work performed by a given collection of third-law force pairs might or might not depend on the specific paths objects take to get from old to new positions. If this total work is **path-independent** for all combinations of initial and final positions in a domain of interest, we can choose to call this total work the negative of the change in the **potential energy** associated with the given collection of third-law force pairs.

$$-\Delta U_{\text{F},1\ldots N} := \Delta W_{\text{F},2\ \text{ON 1}} + \Delta W_{\text{F},1\ \text{ON 2}} + \cdots + \Delta W_{\text{F},N-1\ \text{ON}\ N}$$

$$\Sigma K_i + \sum_{\substack{\text{EXT}\\ \text{ON}\\ \text{SYS}}} \Delta W - \sum_{\substack{\text{SOME}\\ \text{INT}\\ \text{PAIRS}}} \Delta U + \sum_{\substack{\text{OTHER}\\ \text{INT}\\ \text{PAIRS}}} \Delta W = \Sigma K_f$$

One way to begin to describe the motion of a system of particles is to construct simplistic snapshots in which some microscopic details might be neglected (e.g. ignore microscopic lattice vibrations and pretend that a mass distribution is rigid). Total kinetic energy and total potential energy can be expressed using groups of “simplistic” terms and groups of terms that “correct for details.” “Simplistic” terms express energies computed using only simplified snapshots and their dynamics. Simplistic snapshots can fail to depict the precise locations and velocities of individual microscopic particles. Total kinetic and total potential energies computed using simplistic snapshots alone can fail to be accurate. These inaccuracies are made up for by groups of energy terms that “correct for details.”

$$\sum_{\text{SIMPLISTIC}} K_i + \sum_{\substack{\text{CORRECT}\\ \text{FOR}\\ \text{DETAILS}}} K_i + \sum_{\substack{\text{EXT}\\ \text{ON}\\ \text{SYS}}} \Delta W - \sum_{\substack{\text{SOME}\\ \text{INT}\\ \text{SIMPLISTIC}}} \Delta U - \sum_{\substack{\text{SOME}\\ \text{INT}\\ \text{CORRECT}\\ \text{FOR}\\ \text{DETAILS}}} \Delta U + \sum_{\substack{\text{OTHER}\\ \text{INT}\\ \text{PAIRS}}} \Delta W = \sum_{\text{SIMPLISTIC}} K_f + \sum_{\substack{\text{CORRECT}\\ \text{FOR}\\ \text{DETAILS}}} K_f$$

When all the terms that “correct for details” correspond to details that are thought of as “chemical,” it is common to gather the “correct for details” terms together in a single sum of “changes in internal energy.” $\Sigma\Delta W_{\text{OUF}}$ stands for the sum of work otherwise unaccounted for.

$$\overbrace{\Sigma K_i + \Sigma U_{\text{G},i} + \Sigma U_{\text{S},i}}^{\Sigma ME_{\text{SYS},i}} + \Sigma\Delta W_{\text{OUF}} = \overbrace{\Sigma K_f + \Sigma U_{\text{G},f} + \Sigma U_{\text{S},f}}^{\Sigma ME_{\text{SYS},f}} + \Sigma\Delta U_{\text{INT}}$$

For a rigid system of N particles, each particle has same α

$$I_1\alpha = \sum_{\text{ON 1}} \tau$$
$$I_2\alpha = \sum_{\text{ON 2}} \tau$$
$$\vdots$$
$$I_N\alpha = \sum_{\text{ON}\ N} \tau$$

$$\left(\sum I\right)\alpha = \sum_{\text{ON 1}} \tau + \sum_{\text{ON 2}} \tau + \cdots + \sum_{\text{ON}\ N} \tau$$

$$\left(\sum I\right)\alpha = \sum_{\substack{2,\ldots,N\\ \text{ON 1}}} \tau + \sum_{\substack{\text{EXT}\\ \text{ON 1}}} \tau + \sum_{\substack{1,3,\ldots,N\\ \text{ON 2}}} \tau + \sum_{\substack{\text{EXT}\\ \text{ON 2}}} \tau + \cdots + \sum_{\substack{1,\ldots,N-1\\ \text{ON}\ N}} \tau + \sum_{\substack{\text{EXT}\\ \text{ON}\ N}} \tau$$

There is no known force law that generates Newton’s third law interaction force pairs that do not share a common line of action. Using this fact and the above illustration, we can argue that all internal torque pairs cancel.

$$\left(\sum I\right)\alpha = \sum_{\substack{\text{EXT}\\ \text{ON SYS}}} \tau$$

$$\alpha = \frac{\sum \tau}{I}$$

Focus on torque and time	Focus on torque and angular position
$L := I\omega$	$K_{\text{RIGID WITH FIXED SKEWER}} = \frac{1}{2} I_{\substack{\text{ABOUT}\\ \text{FIXED}\\ \text{SKEWER}}} \omega^2$ $\Delta W_{\Sigma\tau} = \left(\sum_{\substack{\text{EXT}\\ \text{ON SYS}}} \tau_{\text{AVG}}\right)\Delta\theta$
$\Sigma L_i + \left(\sum_{\substack{\text{EXT}\\ \text{ON SYS}}} \tau_{\text{AVG}}\right)\Delta t = \Sigma L_f$	$K_{\text{R.W.F.S.},i} + \Delta W_{\Sigma\tau} = K_{\text{R.W.F.S.},f}$