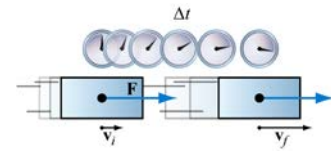


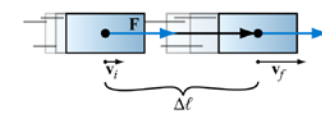
Newton's second law can be re-expressed

$$\vec{a} = \frac{\sum \vec{F}}{m}$$

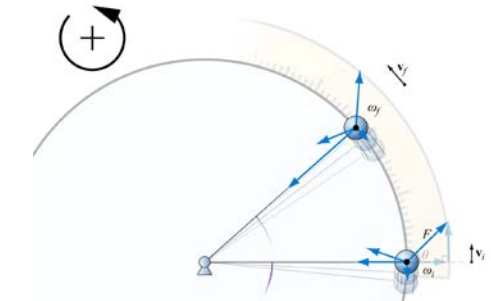
Impulse-momentum theorem (focus on force and time)



Work-energy theorem (focus on force and position)



Rotational formulation for Newton's second law (change coordinate system)



Algebra-based physics

Consider time-averaged force

$$\begin{aligned} \vec{a}_{\text{AVG}} &= \frac{\sum \vec{F}_{\text{AVG}}}{m} \\ \frac{\sum \vec{F}_{\text{AVG}}}{m} &= \frac{\Delta \vec{v}}{\Delta t} \\ \sum \vec{F}_{\text{AVG}} \Delta t &= m \Delta \vec{v} \\ &= m(\vec{v}_f - \vec{v}_i) \\ &= m\vec{v}_f - m\vec{v}_i \\ \sum_{\Delta t_{\text{SF}}} \vec{F}_{\text{AVG}} \Delta t &= \Delta \left(\frac{m\vec{v}}{1} \right) \end{aligned}$$

Calculus-based physics

$$\begin{aligned} \frac{\sum \vec{F}}{m} &= \frac{d\vec{v}}{dt} \\ \int_{t=i}^{t=f} \sum \vec{F} dt &= \int_{\vec{v}=\vec{v}_i}^{\vec{v}=\vec{v}_f} m d\vec{v} \\ &= m \int_{\vec{v}=\vec{v}_i}^{\vec{v}=\vec{v}_f} d\vec{v} \\ &= m(\vec{v}_f - \vec{v}_i) \\ &= m\vec{v}_f - m\vec{v}_i \\ \int_{t=i}^{t=f} \sum \vec{F} dt &= \Delta \left(\frac{m\vec{v}}{1} \right) \end{aligned}$$

Algebra-based physics

Consider uniform acceleration in 1-d

$$\begin{aligned} v_f^2 + 2a\Delta\ell &= v_i^2 \\ v_f^2 + 2\left(\frac{\sum F}{m}\right)\Delta\ell &= v_i^2 \\ \frac{1}{2}mv_f^2 + \frac{(\sum F)\Delta\ell}{\Delta W_{\text{SF}}} &= \frac{1}{2}mv_i^2 + \frac{KE_f}{KE_i} \end{aligned}$$

Calculus-based physics

$$\begin{aligned} \frac{d\vec{v}}{dt} &= \frac{\sum \vec{F}}{m} \\ \int_{\vec{v}=\vec{v}_i}^{\vec{v}=\vec{v}_f} \sum \vec{F} \cdot d\vec{\ell} &= \int_{\vec{v}=\vec{v}_i}^{\vec{v}=\vec{v}_f} m \frac{d\vec{v}}{dt} \cdot d\vec{\ell} \\ &= \int_{\vec{v}=\vec{v}_i}^{\vec{v}=\vec{v}_f} m d\vec{v} \cdot \frac{d\vec{\ell}}{dt} \\ &= \int_{\vec{v}=\vec{v}_i}^{\vec{v}=\vec{v}_f} m d\vec{v} \cdot \vec{v} \\ &= \int_{\vec{v}=\vec{v}_i}^{\vec{v}=\vec{v}_f} m \frac{d(\vec{v} \cdot \vec{v})}{2} \\ &= \frac{1}{2}m \int_{v=v_i}^{v=v_f} d(v^2) \\ \int_{\vec{v}=\vec{v}_i}^{\vec{v}=\vec{v}_f} \sum \vec{F} \cdot d\vec{\ell} &= \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2 \\ \Delta W_{\text{SF}} &= \frac{KE_f}{KE_i} \end{aligned}$$

Algebra-based physics

$$\begin{aligned} a_{\text{IN}}(-r) + a_{\text{TAN}}\hat{t} &= \frac{\sum F_{\text{IN}}(-r) + \sum F_{\text{TAN}}\hat{t}}{m} \\ a_{\text{IN}}(-r) + r\alpha\hat{t} &= \frac{\sum F_{\text{IN}}(-r) + \sum F_{\text{TAN}}\hat{t}}{m} \\ r\alpha &= \frac{\sum F_{\text{TAN}}}{m} \\ \alpha &= \frac{\sum F_{\text{TAN}}}{mr} \\ \alpha &= \frac{\sum rF_{\text{TAN}}}{mr^2} \\ \alpha &= \frac{\sum rF \sin \theta}{I} \end{aligned}$$

$$\begin{aligned} \Delta \vec{J}_{\text{SF}} &= \Delta \vec{p} \\ \vec{p}_i + \sum_{\text{SF}} \Delta \vec{J}_{\text{F}} &= \vec{p}_f \end{aligned}$$

$$\begin{aligned} \Delta W_{\text{SF}} &= \Delta KE \\ KE_i + \Delta W_{\text{SF}} &= KE_f \end{aligned}$$

$$\alpha = \frac{\sum \tau}{I}$$

For system of N particles

$$\begin{aligned} \vec{p}_{1,i} + \Delta \vec{J}_{\text{SF ON } 1} &= \vec{p}_{1,f} \\ \vec{p}_{2,i} + \Delta \vec{J}_{\text{SF ON } 2} &= \vec{p}_{2,f} \\ &\vdots \\ \vec{p}_{N,i} + \Delta \vec{J}_{\text{SF ON } N} &= \vec{p}_{N,f} \\ \Sigma \vec{p}_i + \Sigma \Delta \vec{J}_{\text{SF}} &= \Sigma \vec{p}_f \\ \Sigma \vec{p}_i + \sum_{\text{EXT ON SYS}} \Delta \vec{J} + \sum_{\text{INT}} \Delta \vec{J} &= \Sigma \vec{p}_f \end{aligned}$$

Internal Newton's third law interaction force pairs have opposite directions. The magnitude vs. time plots for each force from a third law pair is identical. Thus, pairs of impulses corresponding to third law force pairs cancel out.

$$\Sigma \vec{p}_i + \sum_{\text{EXT ON SYS}} \Delta \vec{J}_{\text{SF,EXT ON SYS}} = \Sigma \vec{p}_f$$

For system of N particles

$$\begin{aligned} KE_{1,i} + \Delta W_{\text{SF ON } 1} &= KE_{1,f} \\ KE_{2,i} + \Delta W_{\text{SF ON } 2} &= KE_{2,f} \\ &\vdots \\ KE_{N,i} + \Delta W_{\text{SF ON } N} &= KE_{N,f} \\ \Sigma KE_i + \Sigma \Delta W_{\text{SF}} &= \Sigma KE_f \\ \Sigma KE_i + \sum_{\text{EXT ON SYS}} \Delta W + \sum_{\text{CONS}} \Delta W + \sum_{\text{INT NC}} \Delta W &= \Sigma KE_f \end{aligned}$$

Sums over internal works do not necessarily vanish. Even though forces occur in "third law" pairs, the corresponding pairs of works do not necessarily have equal magnitudes.

$$\Sigma KE_i + \sum_{\text{EXT ON SYS}} \Delta W - \sum_{\text{INT CONS}} \Delta U - \sum_{\text{INT NC}} \Delta U = \Sigma KE_f$$

Depending on how you do bookkeeping, the terms you write inside $\sum_{\text{CONS}} \Delta U$ might not 1:1 correspond to the terms you write inside $\sum_{\text{CONS}} \Delta W$.

$$\Sigma KE_i + \sum_{\text{EXT ON SYS}} \Delta W - \sum_{\text{INT CONS}} \Delta U_{\text{G}} - \sum_{\text{INT CONS}} \Delta U_{\text{S}} - \sum_{\text{INT NC}} \Delta U = \Sigma KE_f$$

$$\Sigma KE_i + \sum_{\text{EXT ON SYS}} \Delta U_{\text{G},i} + \sum_{\text{EXT ON SYS}} \Delta U_{\text{S},i} + \sum_{\text{EXT ON SYS}} \Delta W_{\text{F}} = \Sigma KE_f + \sum_{\text{EXT ON SYS}} \Delta U_{\text{G},f} + \sum_{\text{EXT ON SYS}} \Delta U_{\text{S},f} + \Sigma \Delta U_{\text{INT}}$$

For a rigid system of N particles, each particle has same α

$$\begin{aligned} I_1 \alpha &= \sum_{\text{ON } 1} \tau \\ I_2 \alpha &= \sum_{\text{ON } 2} \tau \\ &\vdots \\ I_N \alpha &= \sum_{\text{ON } N} \tau \\ (\Sigma I) \alpha &= \sum_{\text{ON } 1} \tau + \sum_{\text{ON } 2} \tau + \dots + \sum_{\text{ON } N} \tau \\ (\Sigma I) \alpha &= \sum_{\text{ON } 1} \tau + \sum_{\text{EXT ON } 1} \tau + \sum_{\text{EXT ON } 2} \tau + \dots + \sum_{\text{EXT ON } N} \tau + \sum_{\text{EXT ON } N} \tau \end{aligned}$$

There is no known force law that generates Newton's third law interaction force pairs that do not share a common line of action. Using this fact and the above illustration, we can argue that all internal torque pairs cancel.

$$\begin{aligned} (\Sigma I) \alpha &= \sum_{\text{EXT ON SYS}} \tau \\ \alpha &= \frac{\sum \tau}{I} \end{aligned}$$

Focus on torque and time

$$L := I\omega$$

Focus on torque and angular position

$$\begin{aligned} KE_{\text{RIGID WITH FIXED A.O.R.}} &= \frac{1}{2} I_{\text{ABOUT FIXED SKEWER}} \omega^2 \\ \Delta W_{\text{SF}} &= \left(\sum_{\text{EXT ON SYS}} \tau_{\text{AVG}} \right) \Delta \theta \end{aligned}$$

$$\Sigma L_i + \left(\sum_{\text{EXT ON SYS}} \tau_{\text{AVG}} \right) \Delta t = \Sigma L_f$$

$$KE_{\text{R.W.F.A.O.R.},i} + \Delta W_{\text{SF}} = KE_{\text{R.W.F.A.O.R.},f}$$