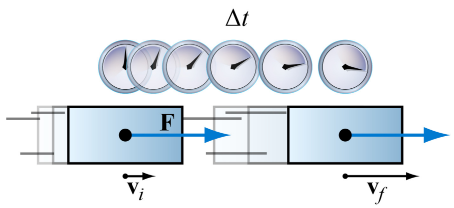
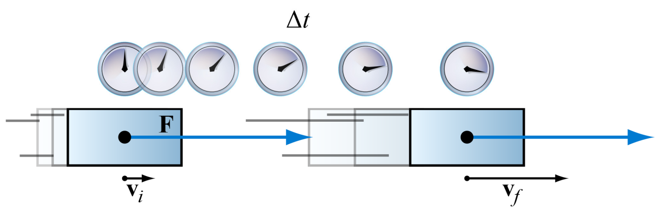
A net force can deliver an impulse that contributes to a change in $m\vec{\mathbf{v}}$

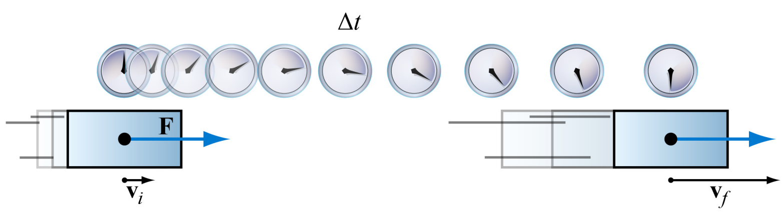
How much can I change the $\vec{\mathbf{v}}$ of an object of mass m by applying a constant force during some interval of time?



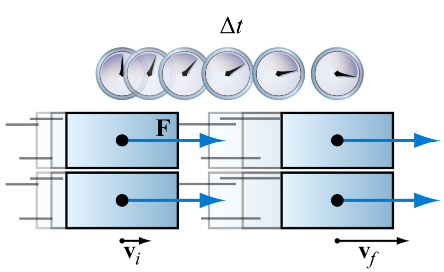
$$\Delta\vec{\mathbf{v}} \neq \vec{\mathbf{0}}$$



$$\uparrow \left|\vec{\mathbf{F}}\right| \Rightarrow \uparrow \left|\Delta\vec{\mathbf{v}}\right|$$



$$\uparrow \Delta t \Rightarrow \uparrow \left|\Delta\vec{\mathbf{v}}\right|$$



$$\uparrow \left|\sum\vec{\mathbf{F}}\right| \Leftarrow \uparrow m$$

Deduced relationship

$$\begin{aligned}\left(\sum\vec{\mathbf{F}}\right)\Delta t &= m\Delta\vec{\mathbf{v}}\\ &= m\left(\vec{\mathbf{v}}_f - \vec{\mathbf{v}}_i\right)\\ &= m\vec{\mathbf{v}}_f - m\vec{\mathbf{v}}_i\\ &= \Delta(m\vec{\mathbf{v}})\end{aligned}$$

Vocabulary

Momentum

$$\vec{\mathbf{p}} := m\vec{\mathbf{v}}$$

Impulse delivered by a force

$$\Delta\vec{\mathbf{J}}_{\mathrm{F}} := \vec{\mathbf{F}}_{\mathrm{AVG}}\Delta t$$

Impulse momentum-theorem

$$\vec{\mathbf{p}}_i + \overbrace{\sum_{\mathrm{F}} \Delta\vec{\mathbf{J}}_{\mathrm{F}}}^{\Delta\vec{\mathbf{J}}_{\Sigma\vec{\mathbf{F}}}} = \vec{\mathbf{p}}_f$$

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Impulse delivered by a varying force

Consider the impulse delivered by a one-dimensional force of varying strength. Allow increments of time to be short enough so that, for each increment, the force is roughly constant.

$$\Delta J_{\mathrm{F},i} \approx F_i \Delta t$$

The total impulse delivered during a finite interval of time

$$\Delta J_{\mathrm{F}} \approx \sum_i F_i \Delta t$$

is the signed area "under" the plot of F vs. t.