Parallels between momentum and energy (algebra-based physics)

Impulse and momentum

The **momentum** of an object is the product of the object's mass and the object's velocity.

$$\vec{\mathbf{p}} := m\vec{\mathbf{v}}$$

The amount of **impulse** delivered is proportional both to the force applied and the duration of time during which the force is applied.

$$\Delta\vec{\mathbf{J}}_\mathrm{F} := \vec{\mathbf{F}}_\mathrm{AVG}\,\Delta t \qquad \begin{aligned}\Delta J_{\mathrm{F},x} &= F_{x,\mathrm{AVG}}\,\Delta t \\ \Delta J_{\mathrm{F},y} &= F_{y,\mathrm{AVG}}\,\Delta t\end{aligned} \qquad \Delta J_{\mathrm{F},x,y} = \begin{array}{c}\text{Signed area}\\ \text{under graph of}\\ F_{x,y}\text{ vs. }t\end{array}$$

The total impulse delivered by the net force acting on an object during a process equals the change in the object's momentum.

$$\vec{\mathbf{p}}_i + \overbrace{\sum_\mathrm{F}\Delta\vec{\mathbf{J}}_\mathrm{F}}^{\Delta\vec{\mathbf{J}}_{\Sigma\vec{\mathbf{F}}}} = \vec{\mathbf{p}}_f$$

The average rate at which impulse is delivered (equivalently, the rate at which momentum is being changed) by a **force** equals the average of that force.

$$\vec{\mathbf{F}}_\mathrm{AVG} = \frac{\Delta\vec{\mathbf{J}}_\mathrm{F}}{\Delta t} = \frac{\Delta\vec{\mathbf{p}}_\mathrm{F}}{\Delta t}$$

In the absence of a net force from external sources, the **total momentum** of a system is **conserved**.

$$\Sigma\vec{\mathbf{P}}_i + \overbrace{\sum_{\substack{\mathrm{EXT}\\ \mathrm{ON\ SYS}}}\Delta\vec{\mathbf{J}}_\mathrm{F}}^{\Delta\vec{\mathbf{J}}_{\Sigma\vec{\mathbf{F}},\mathrm{EXT\ ON\ SYS}}} = \Sigma\vec{\mathbf{P}}_f$$

Work and energy

The **kinetic energy** of an object is half the product of the object's mass and the square of the object's speed.

$$KE := \frac{1}{2}mv^2$$

The amount of **work** performed is proportional to the strength of the force applied and the parallel displacement through which the object is pushed.

$$\begin{aligned}\Delta W_\mathrm{F} &:= F_{\parallel,\mathrm{AVG}}\Delta\ell \\ &= (F\cos\theta)_\mathrm{AVG}\,\Delta\ell\end{aligned} \qquad \Delta W_\mathrm{F} = \begin{array}{c}\text{Signed area}\\ \text{under graph of}\\ F_\parallel\text{ vs. }\ell\end{array} \qquad [W] = \mathrm{N}\cdot\mathrm{m} = \mathrm{J}$$

The total work performed by the net force acting on an object during a process equals the change in the object's kinetic energy.

$$KE_i + \overbrace{\sum_\mathrm{F}\Delta W_\mathrm{F}}^{\Delta W_{\Sigma\vec{\mathbf{F}}}} = KE_f$$

The average rate at which work is performed by a force over time is the average **power** delivered by that force.

$$P_{\mathrm{F,AVG}} := \frac{\Delta W_\mathrm{F}}{\Delta t} \qquad P_\mathrm{F} = (F\cos\theta)v \qquad [P] = \frac{\mathrm{J}}{\mathrm{s}} = \mathrm{W}$$

The work that a given force would perform on an object along a path from the object's present position to a **reference position** might be independent of path. In such a case, this work is called the **potential energy** (associated with the force) at the object's current position.

$$\Delta U_\mathrm{F} := -\Delta W_\mathrm{F} \qquad F_{x,\mathrm{AVG}} = -\frac{\Delta U_\mathrm{F}}{\Delta x} \qquad \Delta U_\mathrm{G} = mg\Delta h \qquad \Delta U_\mathrm{S} = \frac{1}{2}k(\Delta x)^2$$

In the absence of net work from external sources, the **total energy** of a system is **conserved**. The amount of loss of a force's potential energy equals the amount of work that that force does on an object. Work done on an object equals the gain in that object's kinetic energy.

$$\overbrace{\Sigma KE_i + \Sigma U_{\mathrm{G},i} + \Sigma U_{\mathrm{S},i}}^{\Sigma ME_{\mathrm{SYS},i}} + \sum_{\substack{\mathrm{EXT}\\ \mathrm{ON\ SYS}}}\Delta W_\mathrm{F} = \overbrace{\Sigma KE_f + \Sigma U_{\mathrm{G},f} + \Sigma U_{\mathrm{S},f}}^{\Sigma ME_{\mathrm{SYS},f}} + \Sigma\Delta U_\mathrm{INT}$$