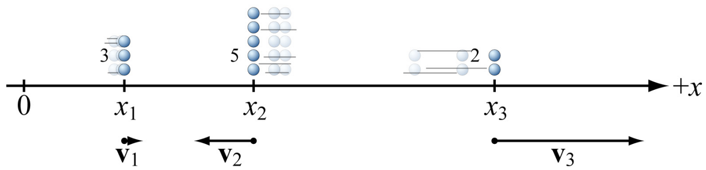
Center of mass

Mass-averaged position



$$x_{\text{C.O.M.}} = \frac{\overbrace{x_1 + x_1 + x_1}^{\text{3 copies}} + \overbrace{x_2 + \cdots + x_2}^{\text{5 copies}} + \overbrace{x_3 + x_3}^{\text{2 copies}}}{10}$$

$$x_{\text{C.O.M.}} = \frac{3}{10}x_1 + \frac{5}{10}x_2 + \frac{2}{10}x_3$$

$$x_{\text{C.O.M.}} := \frac{m_1}{M}x_1 + \frac{m_2}{M}x_2 + \cdots + \frac{m_N}{M}x_N$$

If $\sum_{\text{EXT ON SYS}} F_x = 0$ and $v_{x,\text{C.O.M.},i} = 0$,

$$x_{\text{C.O.M.},i} = x_{\text{C.O.M.},f}$$

Neatly and graphically represent situation(s)

- Draw before and after situations
- Choose reference position and positive direction

Graphically represent quantities and their relationships

- Specify a marked position on each object (if possible, choose markers that are initially at the same position)

Identify relevant allowed starting point (in)equation(s)

- Relative displacement formula: $x_{\text{A,C}} = x_{\text{A,B}} + x_{\text{B,C}}$
- Constancy of center of mass: $x_{\text{C.O.M.},i} = x_{\text{C.O.M.},f}$

Analyze

Communicate

Problem type: Person walks on boat, causing it to slide across water

Mass-averaged velocity

$$v_{x,\text{C.O.M.}} = \frac{\Delta x_{\text{C.O.M.}}}{\Delta t} = \frac{x_{\text{C.O.M.},f} - x_{\text{C.O.M.},i}}{\Delta t}$$

$$v_{x,\text{C.O.M.}} = \frac{\left(\frac{m_1}{M}x_{1,f} + \cdots + \frac{m_N}{M}x_{N,f}\right) - \left(\frac{m_1}{M}x_{1,i} + \cdots + \frac{m_N}{M}x_{N,i}\right)}{\Delta t}$$

$$v_{x,\text{C.O.M.}} = \frac{m_1}{M}\frac{\Delta x_1}{\Delta t} + \frac{m_2}{M}\frac{\Delta x_2}{\Delta t} + \cdots + \frac{m_N}{M}\frac{\Delta x_N}{\Delta t}$$

$$v_{x,\text{C.O.M.}} = \frac{m_1}{M}v_{x,1} + \frac{m_2}{M}v_{x,2} + \cdots + \frac{m_N}{M}v_{x,N}$$

is the velocity of a particle with mass and momentum equal to the total mass and total momentum, respectively, of the system

$$v_{x,\text{C.O.M.}} = \frac{m_1 v_{x,1} + m_2 v_{x,2} + \cdots + m_N v_{x,N}}{M}$$

$$v_{x,\text{C.O.M.}} = \frac{p_{x,1} + p_{x,2} + \cdots + p_{x,N}}{M}$$

$$v_{x,\text{C.O.M.}} = \frac{\sum p_x}{M}$$

$$M v_{x,\text{C.O.M.}} = \sum p_x$$

In the C.O.M. frame, $v_{x,\text{C.O.M.}} = 0$ and $\sum p_x = 0$.

Problem type: Going into C.O.M. frame to solve elastic collisions

Mass-averaged acceleration

$$a_{x,\text{C.O.M.}} = \frac{m_1}{M}a_{x,1} + \frac{m_2}{M}a_{x,2} + \cdots + \frac{m_N}{M}a_{x,N}$$

is proportional to net force on system from external sources

$$a_{x,\text{C.O.M.}} = \frac{m_1 a_{x,1} + m_2 a_{x,2} + \cdots + m_N a_{x,N}}{M}$$

$$a_{x,\text{C.O.M.}} = \frac{\sum_{1,\text{EXT}} F_x + \sum_{1,\text{INT}} F_x + \cdots + \sum_{N,\text{EXT}} F_x + \sum_{N,\text{INT}} F_x}{M}$$

$$a_{x,\text{C.O.M.}} = \frac{\sum_{\text{EXT ON SYS}} F_x}{M}$$

Problem type: Locating fragments of projectile that explodes