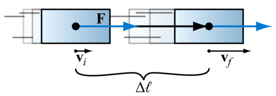
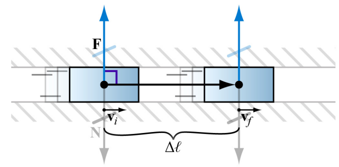
A net force can perform work that contributes to a change in $\frac{1}{2}mv^2$

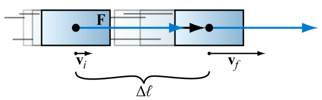
How much can I change the v^2 of an object of mass m by applying a constant force while the object moves through a path length?



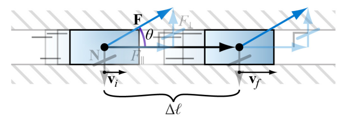
$$\Delta(v^2) \neq 0$$



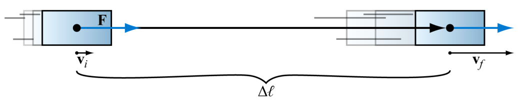
$$\perp \Rightarrow \Delta(v^2) = 0$$



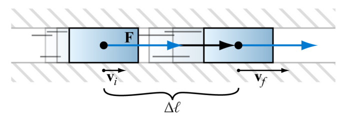
$$\uparrow \left|\vec{\mathbf{F}}\right| \Rightarrow \uparrow |\Delta(v^2)|$$



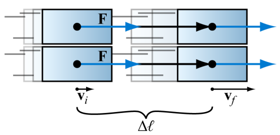
$$\text{oblique} \Rightarrow \text{some } |\Delta(v^2)|$$



$$\uparrow \Delta\ell \Rightarrow \uparrow |\Delta(v^2)|$$



$$\parallel \Rightarrow \max |\Delta(v^2)|$$



$$\uparrow \left|\sum\vec{\mathbf{F}}\right| \Leftarrow \uparrow m$$

Deduced **relationship**

$$\underbrace{\left(\sum F\cos\theta\right)}_{\Sigma F_{\parallel}} \Delta\ell = \frac{1}{2}m\Delta(v^2)$$

$$= \frac{1}{2}m\left(v_f^2 - v_i^2\right)$$

$$= \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2$$

$$= \Delta\left(\frac{1}{2}mv^2\right)$$

Vocabulary

Kinetic energy

$$KE = \frac{1}{2}mv^2$$

Work delivered by a force

$$\Delta W_{\mathrm{F}} := \underbrace{(F\cos\theta)}_{F_{\parallel}} \Delta\ell$$

Work-energy theorem

$$KE_i + \overbrace{\sum_{\mathbf{F}} \Delta W_{\mathrm{F}}}^{\Delta W_{\Sigma\vec{\mathbf{F}}}} = KE_f$$

(for an object having no accessible internal degrees of freedom)

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Work done by a varying force

Consider the work performed by a force of varying strength. Allow increments of path length to be small enough so that, for each increment, the force is roughly constant.

$$\Delta W_{\mathrm{F},i} \approx F_{\parallel,i}\Delta\ell$$

The total work done along a path of finite length

$$\Delta W_{\mathrm{F}} \approx \sum_i F_{\parallel,i}\Delta\ell$$

is the signed area “under” the plot of $F_{\parallel}$ vs. ℓ