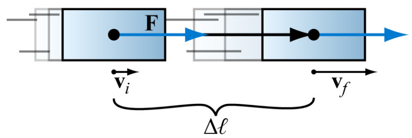
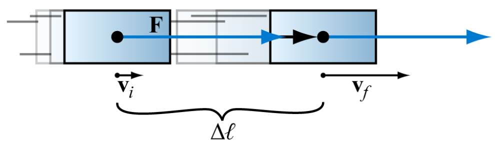
A force can perform work that contributes to a change in $\frac{1}{2}mv^2$

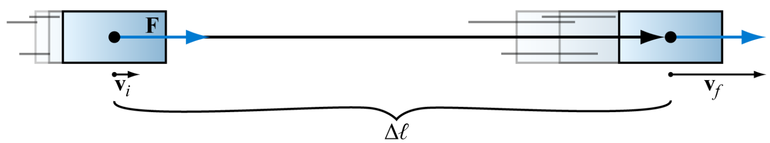
How much can I change the $\frac{1}{2}mv^2$ of an object by applying a constant force while the object moves through some path length?



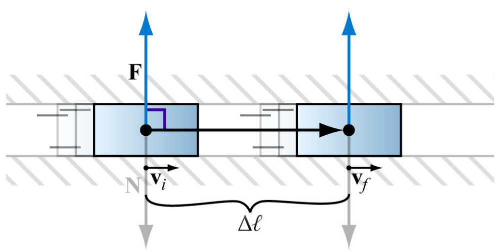
$$\text{change in } \frac{1}{2}mv^2$$



$$\uparrow F \Rightarrow \uparrow \text{change in } \frac{1}{2}mv^2$$

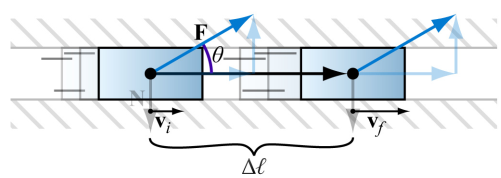


$$\uparrow \Delta\ell \Rightarrow \uparrow \text{change in } \frac{1}{2}mv^2$$



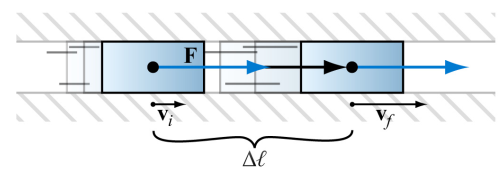
fully $\perp$

$$\Rightarrow \text{fail to change } \frac{1}{2}mv^2$$



partly $\parallel$

$$\Rightarrow \text{change } \frac{1}{2}mv^2 \text{ somewhat}$$



fully $\parallel$

$$\Rightarrow \text{change } \frac{1}{2}mv^2 \text{ a lot}$$

$$KE = \frac{1}{2}mv^2$$

$$\text{Work delivered by a force} = \text{(forcefulness)}(\parallel\text{ness})\text{(distance)} = \text{change in kinetic energy}$$

$$\text{Work delivered by a force} = \Delta W_{\mathrm{F}} := (F\cos\theta)\Delta\ell = \Delta KE$$

A force can perform work that contributes to a change in $\frac{1}{2}mv^2$

Work done by a varying force

Consider the work performed by a force of varying strength. Allow increments of path length to be small enough so that, for each increment, the force is roughly constant.

$$\Delta W_{\mathrm{F},i} \approx F_{\parallel,i}\Delta\ell$$

The total work done along a path of finite length

$$\Delta W_{\mathrm{F}} \approx \sum_i F_{\parallel,i}\Delta\ell$$

is the signed area "under" the plot of $F_\parallel$ vs. ℓ