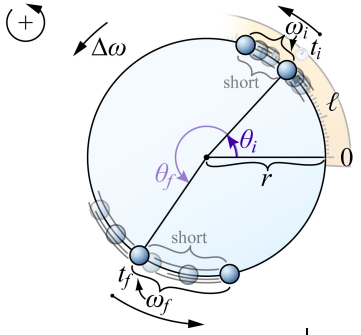
Rotational kinematics and dynamics (algebra-based physics)

Kinematics

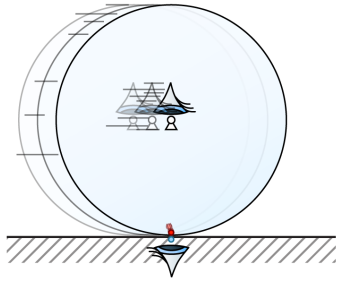


Definitions

Angular	Tangential
$\theta := \dfrac{\ell}{r}$	$\Delta\ell = r\Delta\theta$
$\omega_{\text{AVG}} := \dfrac{\Delta\theta}{\Delta t}$	$v_{\text{TAN}} = r\omega$
$\alpha_{\text{AVG}} := \dfrac{\Delta\omega}{\Delta t}$	$a_{\text{TAN}} = r\alpha$

Relationships

If no slip
$\Delta x_{\text{A.O.R.}} = \Delta\ell$
$v_{\text{A.O.R.}} = v_{\text{TAN}}$
$a_{\text{A.O.R.}} = a_{\text{TAN}}$



Explanation of no-slip condition

$v_{\text{FLOOR REL A.O.R.}} = v_{\text{TAN}}$

$v_{\text{A.O.R. REL FLOOR}} = v_{\text{FLOOR REL A.O.R.}}$

Relationships for UαM

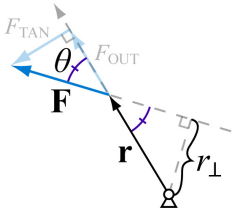
$\theta_i + \omega_{\text{AVG}}\,\Delta t = \theta_f$	α
$\omega_i + \alpha_{\text{AVG}}\,\Delta t = \omega_f$	θ
$\omega_{\text{AVG}} = \dfrac{\omega_i + \omega_f}{2}$	t, θ, α
$\theta_i + \omega_i\Delta t + \dfrac{1}{2}\alpha\Delta t^2 = \theta_f$	
$\omega_i^2 + 2\alpha\Delta\theta = \omega_f^2$	t

Dynamics

Torque

$$\tau_{\text{F}} := r_\perp F$$
$$:= (r\sin\theta)F$$

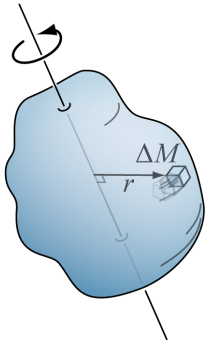
$$\alpha = \frac{\sum\tau}{I}$$



Rotational inertia

$$I_{\substack{\text{RIGID SET}\\\text{OF}\\\text{PARTICLES}}} := \sum_i \Delta M_i r_i^2$$

$$I_{\text{RIGID}} = I_1 + I_2 + I_3 + \cdots$$



Summing torques

1. Draw spatially-extended **free-body diagram** with the **tail** of each force vector anchored at its **point of application**.
2. Draw $+x$ and $+y$ directions.
3. Draw **axis of rotation** and positive **sense of rotation**.
4. Fill in $\sum\tau = I\alpha$, determining the **sign** of each τ by considering whether each force, in isolation, would spin up the object in the ccw or cw direction.

Conservation laws

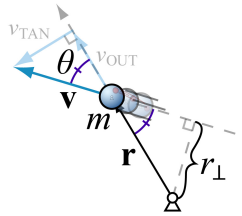
Angular momentum

$$L_{\text{PARTICLES}} := \sum_i \pm_i m_i v_i r_{\perp,i}$$

$$L_{\text{PARTICLE}} = (r\sin\theta)p$$

$$L_{\substack{\text{RIGID}\\\text{FIXED}\\\text{SKEWER}}} = I_{\substack{\text{ABOUT}\\\text{FIXED}\\\text{SKEWER}}}\,\omega$$

$$L_{\text{RIGID}} = L_{\substack{\text{C.O.M.}\\\text{ORBITS}\\\text{A.O.R.}}} + L_{\substack{\text{SPIN}\\\text{ABOUT}\\\text{C.O.M.}}}$$



Angular momentum is constant when the net torque is zero.

$$\Sigma L_i + \left(\sum_{\substack{\text{EXT}\\\text{ON SYS}}} \tau_{\text{AVG}}\right)\Delta t = \Sigma L_f$$

Summing angular momenta

1. Illustrate **before** and **after** situations.
2. Draw **axis of rotation**.
3. Draw positive **sense of rotation**.
4. For each object, determine **sign** of L by determining whether rotation is ccw or cw.

Energy

$$KE_{\text{PARTICLES}} := \sum_i \frac{1}{2}\Delta M_i v_i^2$$

$$KE_{\substack{\text{RIGID}\\\text{FIXED}\\\text{SKEWER}}} = \frac{1}{2} I_{\substack{\text{ABOUT}\\\text{FIXED}\\\text{SKEWER}}}\,\omega^2$$

$$\Delta W_{\tau_{\text{F}}} = \tau_{\text{F,AVG}}\,\Delta\theta$$

$$KE_{\text{RIGID}} = \frac{1}{2} M v_{\text{C.O.M.}}^2 + \frac{1}{2} I_{\substack{\text{ABOUT}\\\text{C.O.M.}}}\,\omega^2$$