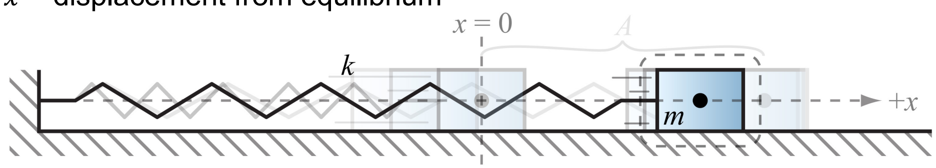
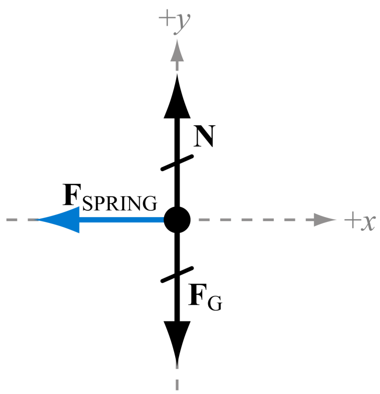
# Simple harmonic motion (SHM) is the one-dimensional shadow of UCM

## Mass on a spring

$x$ – displacement from equilibrium



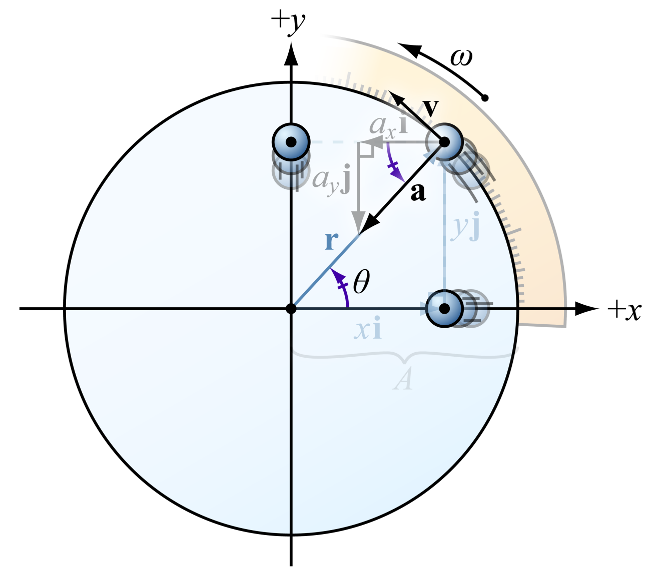

Spring force
Direction: Restoring
Magnitude: ∝ Displacement

$$a_x = \frac{\Sigma F_x}{m}$$

$$a_x = \frac{-k|\Delta x|}{m}$$

$$a_x = \frac{-kx}{m}$$

$$a_x = -\frac{k}{m}x$$




$$\frac{|a_x|}{a_{\mathrm{IN}}} = \frac{|x|}{r}$$

$$|a_x| = \frac{a_{\mathrm{IN}}}{r}|x|$$

$$a_{\mathrm{IN}} = \frac{v_{\mathrm{TAN}}^2}{r} = \omega^2 r$$

$$\frac{a_{\mathrm{IN}}}{r} = \omega^2$$

$$|a_x| = \omega^2 |x|$$

$$a_x = -\omega^2 x$$

$\omega$ – angular velocity of UCM that completes one lap in the same duration of time that SHM of interest completes one cycle of oscillation

$A$ – amplitude (maximum linear or angular distance from equilibrium)

$T$ – period (repetition time)

When magnitude of restoring net force (torque) is ∝ (angular) displacement, $T$ is independent of $A$

$f$ – frequency (oscillations per unit time)

$$\omega^2 = \frac{k}{m}$$

$$x(t) = A\cos\left(\sqrt{\frac{k}{m}}\,t + \delta\right) = A\cos\left(\frac{2\pi}{T}t + \delta\right)$$

$$\omega = \frac{2\pi}{T} \Rightarrow T = \frac{2\pi}{\omega}$$

$$T = \frac{1}{f} = 2\pi\sqrt{\frac{m}{k}}$$