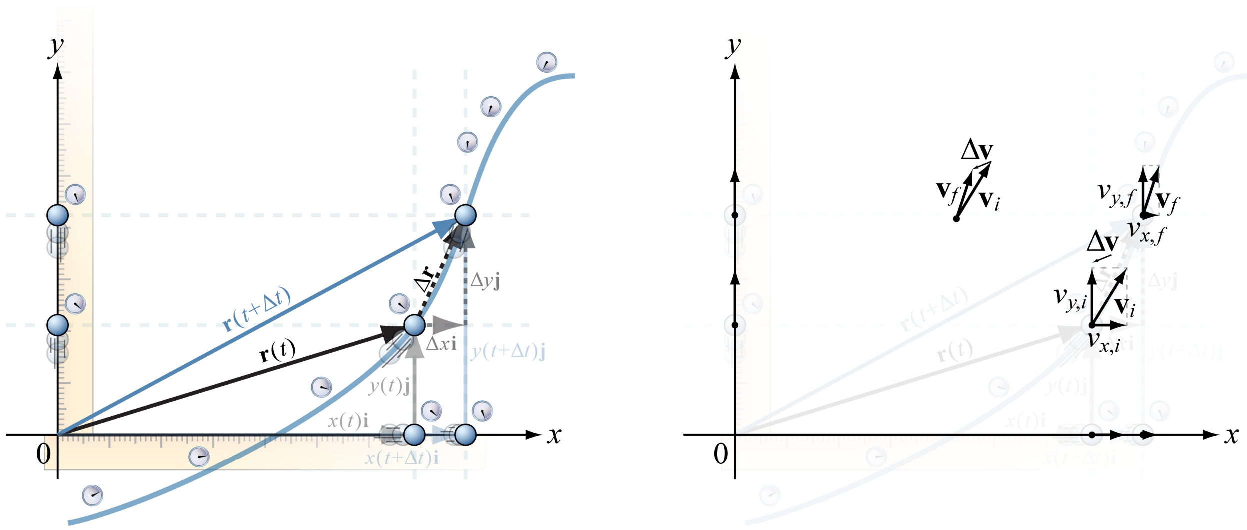
Position function

The motion of an object in two dimensions can be directly analyzed by drawing a motion diagram illustrating velocity vectors with magnitudes and directions drawn to scale.



The motion of an object in two dimensions can also be re-expressed in terms of two 1-d descriptions.

An object moving in multiple dimensions casts “shadows” on the coordinate axes. The shadows undergo simultaneous 1-d motion. The time values that label the actual positions visited by the object in multi-dimensional space are the same time values that label the corresponding shadows on the coordinate axes.

When studying kinematics in multiple dimensions, one-dimensional kinematics relationships can be applied separately to the x coordinate, the y coordinate, etc.

$$\vec{\mathbf{r}}(t) = x(t)\hat{\mathbf{i}} + y(t)\hat{\mathbf{j}}$$

$$x(t_i) + \int_{t_i}^{t_f} v_x(t)\,\mathrm{d}t = x\left(t_f\right)$$

$$v_x(t) = \frac{\mathrm{d}}{\mathrm{d}t} x(t)$$

$$v_x(t_i) + \int_{t_i}^{t_f} a_x(t)\,\mathrm{d}t = v_x\left(t_f\right)$$

$$a_x(t) = \frac{\mathrm{d}^2}{\mathrm{d}t^2} x(t)$$

The definition of speed introduced in the reference sheet for 1-d motion can be extended to motion in multiple dimensions.

$$\text{speed} := |\vec{\mathbf{v}}|$$

Computing velocity function from position function

$$\vec{\mathbf{v}}(t) := \frac{\mathrm{d}}{\mathrm{d}t}\vec{\mathbf{r}}(t)$$

$$= \frac{\mathrm{d}}{\mathrm{d}t}[x(t)\hat{\mathbf{i}} + y(t)\hat{\mathbf{j}}]$$

$$= \frac{\mathrm{d}}{\mathrm{d}t}[x(t)\hat{\mathbf{i}}] + \frac{\mathrm{d}}{\mathrm{d}t}[y(t)\hat{\mathbf{j}}]$$

$$\vec{\mathbf{v}}(t) = \left[\frac{\mathrm{d}}{\mathrm{d}t}x(t)\right]\hat{\mathbf{i}} + \left[\frac{\mathrm{d}}{\mathrm{d}t}y(t)\right]\hat{\mathbf{j}}$$

$$v_x(t)\hat{\mathbf{i}} + v_y(t)\hat{\mathbf{j}} = \left[\frac{\mathrm{d}}{\mathrm{d}t}x(t)\right]\hat{\mathbf{i}} + \left[\frac{\mathrm{d}}{\mathrm{d}t}y(t)\right]\hat{\mathbf{j}}$$

$$v_x(t) = \frac{\mathrm{d}}{\mathrm{d}t}x(t)$$

$$v_y(t) = \frac{\mathrm{d}}{\mathrm{d}t}y(t)$$

Computing acceleration function from velocity function

$$\vec{\mathbf{a}}(t) := \frac{\mathrm{d}}{\mathrm{d}t}\vec{\mathbf{v}}(t)$$

$$= \frac{\mathrm{d}}{\mathrm{d}t}\left(\left[\frac{\mathrm{d}}{\mathrm{d}t}x(t)\right]\hat{\mathbf{i}} + \left[\frac{\mathrm{d}}{\mathrm{d}t}y(t)\right]\hat{\mathbf{j}}\right)$$

$$= \frac{\mathrm{d}}{\mathrm{d}t}\left(\left[\frac{\mathrm{d}}{\mathrm{d}t}x(t)\right]\hat{\mathbf{i}}\right) + \frac{\mathrm{d}}{\mathrm{d}t}\left(\left[\frac{\mathrm{d}}{\mathrm{d}t}y(t)\right]\hat{\mathbf{j}}\right)$$

$$\vec{\mathbf{a}}(t) = \left[\frac{\mathrm{d}^2}{\mathrm{d}t^2}x(t)\right]\hat{\mathbf{i}} + \left[\frac{\mathrm{d}^2}{\mathrm{d}t^2}y(t)\right]\hat{\mathbf{j}}$$

$$a_x(t)\hat{\mathbf{i}} + a_y(t)\hat{\mathbf{j}} = \left[\frac{\mathrm{d}^2}{\mathrm{d}t^2}x(t)\right]\hat{\mathbf{i}} + \left[\frac{\mathrm{d}^2}{\mathrm{d}t^2}y(t)\right]\hat{\mathbf{j}}$$

$$a_x(t) = \frac{\mathrm{d}^2}{\mathrm{d}t^2}x(t) \qquad\qquad a_y(t) = \frac{\mathrm{d}^2}{\mathrm{d}t^2}y(t)$$

Computing position function from velocity function

$$\vec{\mathbf{r}}(t_f) = \vec{\mathbf{r}}(t_i) + \int_{t_i}^{t_f} \vec{\mathbf{v}}(t)\,\mathrm{d}t$$

$$= \vec{\mathbf{r}}(t_i) + \int_{t_i}^{t_f} \left[v_x(t)\hat{\mathbf{i}} + v_y(t)\hat{\mathbf{j}}\right] \mathrm{d}t$$

$$= \vec{\mathbf{r}}(t_i) + \int_{t_i}^{t_f} v_x(t)\hat{\mathbf{i}}\,\mathrm{d}t + \int_{t_i}^{t_f} v_y(t)\hat{\mathbf{j}}\,\mathrm{d}t$$

$$= \vec{\mathbf{r}}(t_i) + \left[\int_{t_i}^{t_f} v_x(t)\,\mathrm{d}t\right]\hat{\mathbf{i}} + \left[\int_{t_i}^{t_f} v_y(t)\,\mathrm{d}t\right]\hat{\mathbf{j}}$$

$$= x(t_i)\hat{\mathbf{i}} + y(t_i)\hat{\mathbf{j}} + \left[\int_{t_i}^{t_f} v_x(t)\,\mathrm{d}t\right]\hat{\mathbf{i}} + \left[\int_{t_i}^{t_f} v_y(t)\,\mathrm{d}t\right]\hat{\mathbf{j}}$$

$$\vec{\mathbf{r}}(t_f) = \left[x(t_i) + \int_{t_i}^{t_f} v_x(t)\,\mathrm{d}t\right]\hat{\mathbf{i}} + \left[y(t_i) + \int_{t_i}^{t_f} v_y(t)\,\mathrm{d}t\right]\hat{\mathbf{j}}$$

$$x(t_f)\hat{\mathbf{i}} + y(t_f)\hat{\mathbf{j}} = \left[x(t_i) + \int_{t_i}^{t_f} v_x(t)\,\mathrm{d}t\right]\hat{\mathbf{i}} + \left[y(t_i) + \int_{t_i}^{t_f} v_y(t)\,\mathrm{d}t\right]\hat{\mathbf{j}}$$

$$x(t_f) = x(t_i) + \int_{t_i}^{t_f} v_x(t)\,\mathrm{d}t \qquad y(t_f) = y(t_i) + \int_{t_i}^{t_f} v_y(t)\,\mathrm{d}t$$

Computing velocity function from acceleration function

$$\vec{\mathbf{v}}(t_f) = \vec{\mathbf{v}}(t_i) + \int_{t_i}^{t_f} \vec{\mathbf{a}}(t)\,\mathrm{d}t$$

$$= \vec{\mathbf{v}}(t_i) + \int_{t_i}^{t_f} \left[a_x(t)\hat{\mathbf{i}} + a_y(t)\hat{\mathbf{j}}\right] \mathrm{d}t$$

$$= \vec{\mathbf{v}}(t_i) + \int_{t_i}^{t_f} a_x(t)\hat{\mathbf{i}}\,\mathrm{d}t + \int_{t_i}^{t_f} a_y(t)\hat{\mathbf{j}}\,\mathrm{d}t$$

$$= \vec{\mathbf{v}}(t_i) + \left[\int_{t_i}^{t_f} a_x(t)\,\mathrm{d}t\right]\hat{\mathbf{i}} + \left[\int_{t_i}^{t_f} a_y(t)\,\mathrm{d}t\right]\hat{\mathbf{j}}$$

$$= v_x(t_i)\hat{\mathbf{i}} + v_y(t_i)\hat{\mathbf{j}} + \left[\int_{t_i}^{t_f} a_x(t)\,\mathrm{d}t\right]\hat{\mathbf{i}} + \left[\int_{t_i}^{t_f} a_y(t)\,\mathrm{d}t\right]\hat{\mathbf{j}}$$

$$\vec{\mathbf{v}}(t_f) = \left[v_x(t_i) + \int_{t_i}^{t_f} a_x(t)\,\mathrm{d}t\right]\hat{\mathbf{i}} + \left[v_y(t_i) + \int_{t_i}^{t_f} a_y(t)\,\mathrm{d}t\right]\hat{\mathbf{j}}$$

$$v_x(t_f)\hat{\mathbf{i}} + v_y(t_f)\hat{\mathbf{j}} = \left[v_x(t_i) + \int_{t_i}^{t_f} a_x(t)\,\mathrm{d}t\right]\hat{\mathbf{i}} + \left[v_y(t_i) + \int_{t_i}^{t_f} a_y(t)\,\mathrm{d}t\right]\hat{\mathbf{j}}$$

$$v_x(t_f) = v_x(t_i) + \int_{t_i}^{t_f} a_x(t)\,\mathrm{d}t \qquad v_y(t_f) = v_y(t_i) + \int_{t_i}^{t_f} a_y(t)\,\mathrm{d}t$$