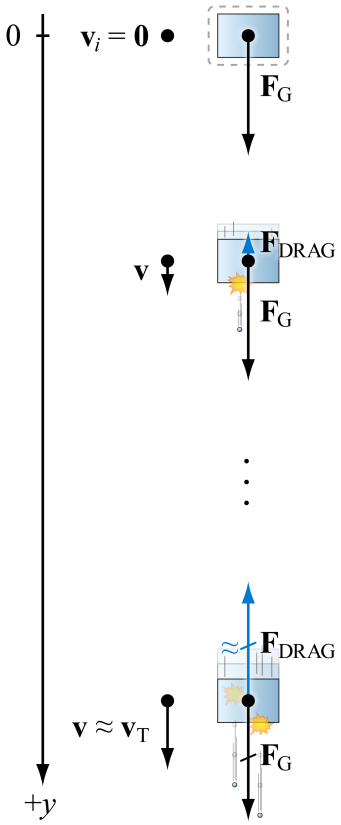
Integrating to find speed vs. time when a drag is present



$$a_y = \frac{\sum F_y}{m}$$

$$\frac{\mathrm{d}v_y}{\mathrm{d}t} = \frac{+F_G - F_{DRAG}}{m}$$

Suppose $F_{DRAG} = b|v|$.

$$\frac{\mathrm{d}v_y}{\mathrm{d}t} = \frac{+mg - b|v|}{m} = g - \frac{b}{m}v_y$$

Early times (just after releasing object)

$$\frac{\mathrm{d}v_y}{\mathrm{d}t} = g - \frac{b}{m}\overbrace{v_y}^{0}$$

Intermediate time t

$$\frac{\mathrm{d}v_y}{\mathrm{d}t} = g - \frac{b}{m}v_y$$

$$\int_{v_y=v_{y,i}}^{v_y=v_{y,f}} \frac{1}{g - \frac{b}{m}v_y}\,\mathrm{d}v_y = \int_{t=t_i}^{t=t_f} \mathrm{d}t$$

$$-\frac{m}{b}\left[\ln\left|g - \frac{b}{m}v_y\right|\right]_{v_y=v_{y,i}}^{v_y=v_{y,f}} = t_f - t_i$$

Note: $g - \frac{b}{m}v_y > 0$

$$-\frac{m}{b}\ln\left(\frac{g - \frac{b}{m}v_{y,f}}{g - \frac{b}{m}v_{y,i}}\right) = t_f - t_i$$

	Time	y-velocity
Initial	$t_i = 0$	$v_{y,i} = 0$
Final	$t_f = t$	$v_{y,f} = v_y$

$$v_y = \underbrace{\frac{mg}{b}}_{v_T}\left(1 - e^{-\frac{b}{m}t}\right)$$

Late times (long after releasing object)

$$\overbrace{\frac{\mathrm{d}v_y}{\mathrm{d}t}}^{0} = g - \frac{b}{m}\overbrace{v_y}^{v_T}$$

$$v_T = \frac{mg}{b}$$