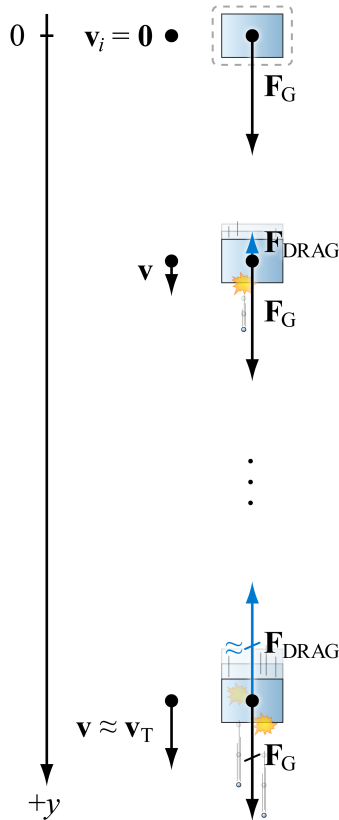


Integrating to find speed vs. time when a drag is present



$$a_y = \frac{\sum F_y}{m}$$

$$\frac{dv_y}{dt} = \frac{+F_G - F_{DRAG}}{m}$$

Suppose $F_{DRAG} = b|v|$.

$$\frac{dv_y}{dt} = \frac{+mg - b|v|}{m} = g - \frac{b}{m}v_y$$

Early times (just after releasing object)

$$\frac{dv_y}{dt} = g - \frac{b}{m} \overset{0}{v_y}$$

Intermediate time t

$$\frac{dv_y}{dt} = g - \frac{b}{m}v_y$$

$$\int_{v_y=v_{y,i}}^{v_y=v_{y,f}} \frac{1}{g - \frac{b}{m}v_y} dv_y = \int_{t=t_i}^{t=t_f} dt$$

$$-\frac{m}{b} \left[\ln \left| g - \frac{b}{m}v_y \right| \right]_{v_y=v_{y,i}}^{v_y=v_{y,f}} = t_f - t_i$$

Note: $g - \frac{b}{m}v_y > 0$

$$-\frac{m}{b} \ln \left(\frac{g - \frac{b}{m}v_{y,f}}{g - \frac{b}{m}v_{y,i}} \right) = t_f - t_i$$

	Time	y-velocity
Initial	$t_i = 0$	$v_{y,i} = 0$
Final	$t_f = t$	$v_{y,f} = v_y$

$$v_y = \underbrace{\frac{mg}{b}}_{v_T} \left(1 - e^{-\frac{b}{m}t} \right)$$

Late times (long after releasing object)

$$\overset{0}{\frac{dv_y}{dt}} = g - \frac{b}{m} \overset{v_T}{v_y}$$

$$v_T = \frac{mg}{b}$$

