Parallels between energy and momentum (calculus-based physics)

Impulse and momentum

The **momentum** of an object is the product of the object's mass and the object's velocity.

$$\vec{\mathbf{p}} := m\vec{\mathbf{v}}$$

The amount of **impulse** delivered is proportional to the force applied and the duration of time during which the force is applied.

$$\mathrm{d}\vec{\mathbf{J}}_\mathrm{F} := \vec{\mathbf{F}}\mathrm{d}t \qquad \int_{t=t_i}^{t=t_f} \mathrm{d}\vec{\mathbf{J}}_\mathrm{F} = \Delta\vec{\mathbf{J}}_\mathrm{F}$$

The total impulse delivered by the net force acting on an object during a process equals the change in the object's momentum.

$$\vec{\mathbf{p}}_i + \overbrace{\sum_\mathrm{F} \Delta\vec{\mathbf{J}}_\mathrm{F}}^{\Delta\vec{\mathbf{J}}_{\Sigma\vec{\mathbf{F}}}} = \vec{\mathbf{p}}_f$$

The instantaneous rate at which impulse is delivered (equivalently, the rate at which momentum is being changed) by a **force** equals the force itself.

$$\vec{\mathbf{F}} = \frac{\mathrm{d}\vec{\mathbf{J}}_\mathrm{F}}{\mathrm{d}t} = \frac{\mathrm{d}\vec{\mathbf{p}}_\mathrm{F}}{\mathrm{d}t}$$

In the absence of a net force from external sources, the **total momentum** of a system is **conserved**.

$$\Sigma\vec{\mathbf{P}}_i + \overbrace{\sum_{\substack{\mathrm{EXT} \\ \mathrm{ON\ SYS}}} \Delta\vec{\mathbf{J}}_\mathrm{F}}^{\Delta\vec{\mathbf{J}}_{\Sigma\vec{\mathbf{F}},\mathrm{EXT\ ON\ SYS}}} = \Sigma\vec{\mathbf{P}}_f$$

Work and energy

The **kinetic energy** of an object is half the product of the object's mass and the square of the object's speed.

$$KE := \frac{1}{2}mv^2$$

The amount of **work** performed is proportional to the strength of the force applied and the parallel displacement through which the object is pushed.

$$\begin{aligned}\mathrm{d}W_\mathrm{F} &:= \vec{\mathbf{F}} \cdot \mathrm{d}\vec{\boldsymbol{\ell}} \\ &= F_\parallel \mathrm{d}\ell \\ &= (F\cos\theta)\mathrm{d}\ell\end{aligned} \qquad \int_{\vec{\mathbf{r}}=\vec{\mathbf{r}}_i}^{\vec{\mathbf{r}}=\vec{\mathbf{r}}_f} \mathrm{d}W_\mathrm{F} = \Delta W_\mathrm{F} \qquad [W] = \mathrm{N}\cdot\mathrm{m} = \mathrm{J}$$

The total work performed by the net force acting on an object during a process equals the change in the object's kinetic energy.

$$KE_i + \overbrace{\sum_\mathrm{F} \Delta W_\mathrm{F}}^{\Delta W_{\Sigma\vec{\mathbf{F}}}} = KE_f$$

The instantaneous rate at which work is performed by a force over time is the instantaneous **power** delivered by that force.

$$P_\mathrm{F} := \frac{\mathrm{d}W_\mathrm{F}}{\mathrm{d}t} \qquad \begin{aligned}P_\mathrm{F} &= \vec{\mathbf{F}}\cdot\vec{\mathbf{v}} \\ &= (F\cos\theta)v\end{aligned} \qquad [P] = \frac{\mathrm{J}}{\mathrm{s}} = \mathrm{W}$$

The work that a given force would perform on an object along a path from the object's present position to a **reference position** might be independent of path. In such a case, this work is called the **potential energy** (associated with the force) at the object's current position.

$$\Delta U_\mathrm{F} := -\Delta W_\mathrm{F} \qquad F_x = -\frac{\mathrm{d}U_\mathrm{F}}{\mathrm{d}x} \qquad \Delta U_\mathrm{G} = mg\Delta h \qquad \Delta U_\mathrm{S} = \frac{1}{2}k(\Delta x)^2$$

In the absence of net work from external sources, the **total energy** of a system is **conserved**. The amount of loss of a force's positional energy equals the amount of work that that force does on an object. Work done on an object equals the gain in that object's kinetic energy.

$$\overbrace{\Sigma KE_i + \Sigma U_{\mathrm{G},i} + \Sigma U_{\mathrm{S},i}}^{\Sigma ME_{\mathrm{SYS},i}} + \sum_{\substack{\mathrm{EXT} \\ \mathrm{ON\ SYS}}} \Delta W_\mathrm{F} = \overbrace{\Sigma KE_f + \Sigma U_{\mathrm{G},f} + \Sigma U_{\mathrm{S},f}}^{\Sigma ME_{\mathrm{SYS},f}} + \Sigma\Delta U_\mathrm{INT}$$