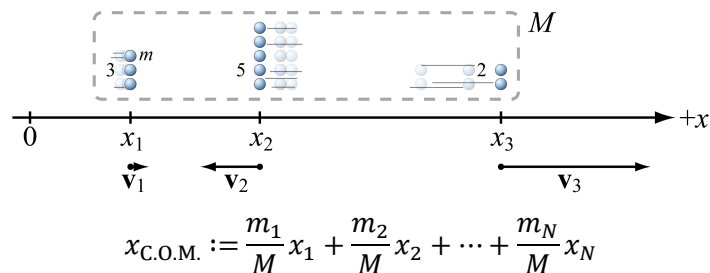


# Mass-averaging by summing over point masses generalizes to integrals

For a **discrete set of masses**, each with known C.O.M.,



Neatly and graphically represent **situation(s)**

Graphically represent **quantities and their relationships**

1. Draw **diagram** of the objects, labeling each object's mass and the position of each object's c.o.m.

Identify relevant allowed starting point (in) **equation(s)**

2. **Substitute** into

$$x_{\text{C.O.M.}} := \frac{m_1 x_1 + m_2 x_2 + \dots + m_3 x_3}{M}$$

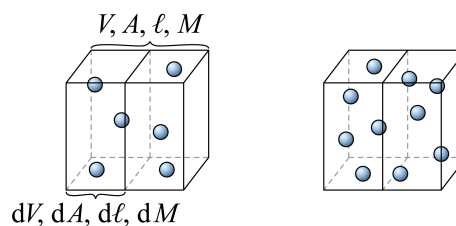
$$y_{\text{C.O.M.}} := \frac{m_1 y_1 + m_2 y_2 + \dots + m_3 y_3}{M}$$

$$z_{\text{C.O.M.}} := \frac{m_1 z_1 + m_2 z_2 + \dots + m_3 z_3}{M}$$

**Analyze**

**Communicate**

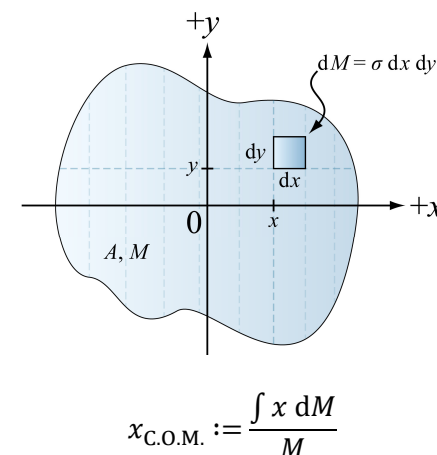
**Mass density**



$$dM = \rho dV$$

For a uniform mass density,

$$\frac{dM}{M} = \frac{dV}{V}$$



Neatly and graphically represent **situation(s)**

1. Draw large **diagram** of object.
2. Label the **boundary functions**.
3. Label the points of **intersection**.

Graphically represent **quantities and their relationships**

4. Draw a small 1-dimensional **differential length element**  $d\ell$ , a small 2-dimensional **differential area element**  $dA$  or a small 3-dimensional **differential volume element**  $dV$  in the interior/bulk of the object.

Identify relevant allowed starting point (in) **equation(s)**

5. Draw an arrow pointing to that differential portion of the object, labeling it with its **differential mass**,  $dM = \lambda d\ell$ ,  $dM = \sigma dA$ , or  $dM = \rho dV$ .
6. **Write** relevant integral expression(s).

$$x_{\text{C.O.M.}} := \frac{\int_{\text{FORM}} x dM}{M} \quad y_{\text{C.O.M.}} := \frac{\int_{\text{FORM}} y dM}{M} \quad z_{\text{C.O.M.}} := \frac{\int_{\text{FORM}} z dM}{M}$$

**Analyze**

7. **Substitute** differential mass element into integral expression(s) and **integrate**.

**Communicate**