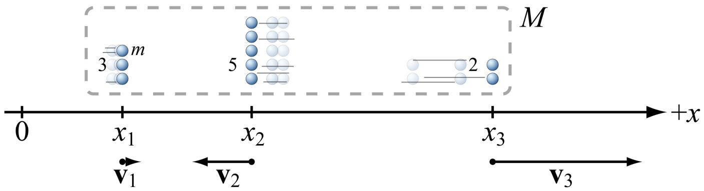
Mass-averaging by summing over point masses generalizes to integrals

For a **discrete set of masses**, each with known C.O.M.,



$$x_{\text{C.O.M.}} := \frac{m_1}{M} x_1 + \frac{m_2}{M} x_2 + \cdots + \frac{m_N}{M} x_N$$

Neatly and graphically represent situation(s)

Graphically represent quantities and their relationships

1. Draw **diagram** of the objects, labeling each object's mass and the position of each object's c.o.m.

Identify relevant allowed starting point (in)equation(s)

2. **Substitute** into

$$x_{\text{C.O.M.}} := \frac{m_1 x_1 + m_2 x_2 + \cdots + m_3 x_3}{M}$$

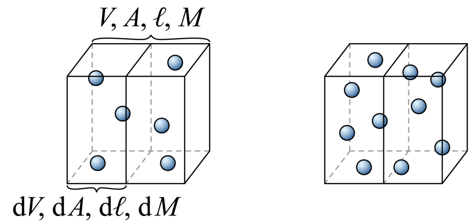
$$y_{\text{C.O.M.}} := \frac{m_1 y_1 + m_2 y_2 + \cdots + m_3 y_3}{M}$$

$$z_{\text{C.O.M.}} := \frac{m_1 z_1 + m_2 z_2 + \cdots + m_3 z_3}{M}$$

Use numbered steps to show REASoNing

Communicate

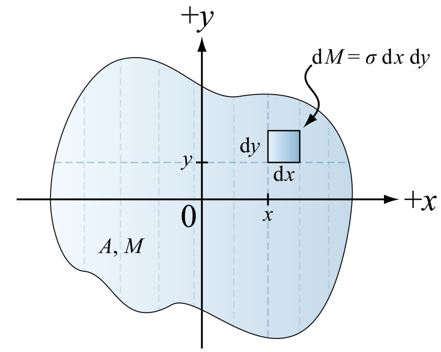
Mass density



$$\mathrm{d}M = \rho\, \mathrm{d}V$$

For a uniform mass density,

$$\frac{\mathrm{d}M}{M} = \frac{\mathrm{d}V}{V}$$



$$x_{\text{C.O.M.}} := \frac{\int x\, \mathrm{d}M}{M}$$

Neatly and graphically represent situation(s)

1. Draw large **diagram** of object.
2. Label the **boundary functions**.
3. Label the points of **intersection**.

Graphically represent quantities and their relationships

4. Draw a small 1-dimensional **differential length element** $\mathrm{d}\ell$, a small 2-dimensional **differential area element** $\mathrm{d}A$ **or** a small 3-dimensional **differential volume element** $\mathrm{d}V$ in the interior/bulk of the object.

Identify relevant allowed starting point (in)equation(s)

5. Draw an arrow pointing to that differential portion of the object, labeling it with its **differential mass**, $\mathrm{d}M = \lambda\, \mathrm{d}\ell$, $\mathrm{d}M = \sigma\, \mathrm{d}A$, or $\mathrm{d}M = \rho\, \mathrm{d}V$.
6. **Write** relevant integral expression(s).

$$x_{\text{C.O.M.}} := \frac{\int_{\text{FORM}} x\, \mathrm{d}M}{M} \qquad y_{\text{C.O.M.}} := \frac{\int_{\text{FORM}} y\, \mathrm{d}M}{M} \qquad z_{\text{C.O.M.}} := \frac{\int_{\text{FORM}} z\, \mathrm{d}M}{M}$$

7. **Substitute** differential mass element into integral expression(s) and **integrate**.

Use numbered steps to show REASoNing

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