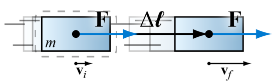
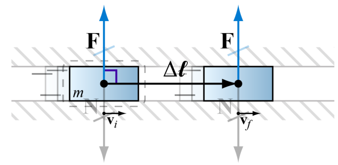
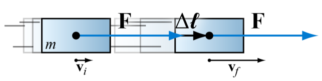
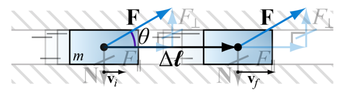
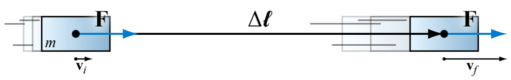
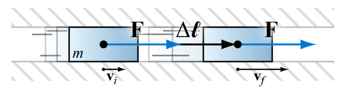
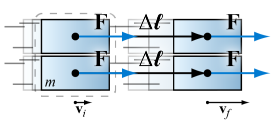
A net force can perform work that contributes to a change in $\frac{1}{2}mv^2$

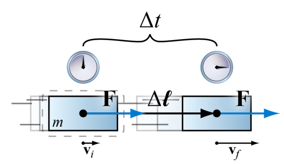
How much can I change the v^2 of an object of mass m by applying a constant force while the object moves through a path length?

	$\Delta(v^2) \neq 0$		$\vec{\mathbf{F}} \perp \Delta\vec{\boldsymbol{\ell}} \Rightarrow \Delta(v^2) = 0$
	$\uparrow \lvert\vec{\mathbf{F}}\rvert \Rightarrow \uparrow \lvert\Delta(v^2)\rvert$		oblique $\Rightarrow$ some $\lvert\Delta(v^2)\rvert$
	$\uparrow \Delta\ell \Rightarrow \uparrow \lvert\Delta(v^2)\rvert$		$\vec{\mathbf{F}} \parallel \Delta\vec{\boldsymbol{\ell}} \Rightarrow \max \lvert\Delta(v^2)\rvert$
	$\uparrow \lvert\sum\vec{\mathbf{F}}\rvert \Leftarrow \uparrow m$		

Deduced **relationship**

$$\underbrace{\left(\sum F\cos\theta\right)}_{\Sigma F_{\parallel}}\Delta\ell = \frac{1}{2}m\Delta(v^2)$$
$$= \frac{1}{2}m\left(v_f^2 - v_i^2\right)$$
$$= \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2$$
$$= \Delta\left(\frac{1}{2}mv^2\right)$$

Vocabulary



Kinetic energy $\quad K := \frac{1}{2}mv^2$

Work delivered by a force $\quad \Delta W_{\mathrm{F}} := \underbrace{(F\cos\theta)}_{F_{\parallel}}\Delta\ell$

Power delivered by a force $\quad P_{\mathrm{F,AVG}} := \frac{\Delta W_{\mathrm{F}}}{\Delta t}$

Work-energy theorem

$$K_i + \overbrace{\sum_{\mathrm{F}} \Delta W_{\mathrm{F}}}^{\Delta W_{\Sigma\vec{\mathrm{F}}}} = K_f$$

(for an object having no accessible internal degrees of freedom)

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Work done by a varying force

Consider the work performed by a force of varying strength. Allow increments of path length to be small enough so that, for each increment, the force is roughly constant.

$$\Delta W_{\mathrm{F},i} \approx F_{\parallel,i}\Delta\ell$$

The total work done along a path of finite length

$$\Delta W_{\mathrm{F}} \approx \sum_i F_{\parallel,i}\Delta\ell$$

is the signed area “under” the plot of $F_\parallel$ vs. ℓ