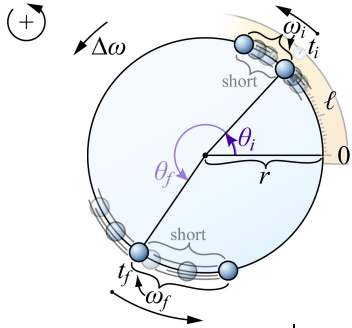


Rotational kinematics and dynamics (calculus-based physics)

Kinematics



Definitions

Angular

$$\theta := \frac{\ell}{r}$$

$$\omega := \frac{d\theta}{dt}$$

$$\alpha := \frac{d\omega}{dt}$$

Tangential

$$\Delta \ell = r \Delta \theta$$

$$v_{TAN} = r \omega$$

$$a_{TAN} = r \alpha$$

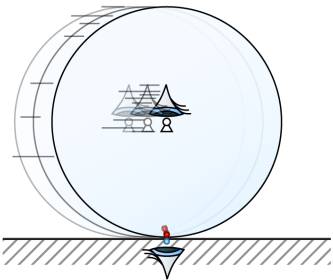
Relationships

If non-slip

$$\Delta x_{A.O.R.} = \Delta \ell$$

$$v_{A.O.R.} = v_{TAN}$$

$$a_{A.O.R.} = a_{TAN}$$



Explanation of no-slip condition

$$v_{FLOOR}^{REL} = v_{TAN}^{REL} \text{ A.O.R.}$$

$$v_{A.O.R.}^{REL} = v_{FLOOR}^{REL} \text{ A.O.R.}$$

Relationships for UαM

$$\theta_i + \omega_{AVG} \Delta t = \theta_f$$

$$\theta_i + \alpha_{AVG} \Delta t = \theta_f$$

$$\omega_{AVG} = \frac{\omega_i + \omega_f}{2}$$

$$\theta_i + \omega_i \Delta t + \frac{1}{2} \alpha \Delta t^2 = \theta_f$$

$$\omega_i^2 + 2\alpha \Delta \theta = \omega_f^2$$

α

θ

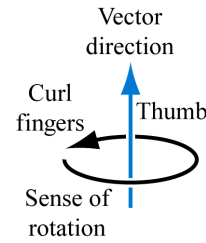
t, θ, α

t

Dynamics

Rotational vectors

RHR

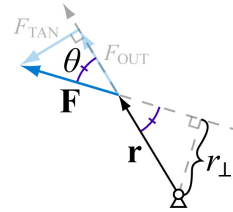


Torque

$$\vec{\tau}_F := \vec{r} \times \vec{F}$$

$$\tau_F = r_{\perp} F = (r \sin \theta) F$$

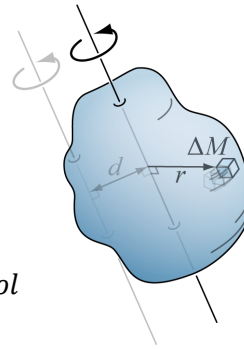
$$\vec{\alpha} = \frac{\sum \vec{\tau}}{I}$$



Rotational inertia

$$I_{RIGID SET OF PARTICLES} := \sum_i \Delta M_i r_i^2$$

$$I_{RIGID CONTINUOUS MASS DISTR.} := \int r^2 dM = \int r^2 \rho dVol$$



$$I_{RIGID PARTS} = I_1 + I_2 + I_3 + \dots$$

$$I_{\parallel} = I_{CM} + Md^2$$

Summing torques

1. Draw spatially-extended **free-body diagram** with the **tail** of each force vector anchored at its **point of application**.
2. Draw +x and +y directions, **axis of rotation**, and positive **sense of rotation**.
3. Fill in $\sum \tau = I\alpha$, determining the **sign** of each τ by considering whether each force, in isolation, would spin up the object in the ccw or cw direction.

Conservation laws

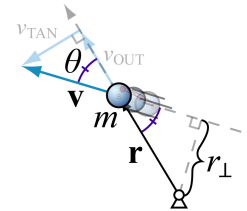
Angular momentum

$$\vec{L}_{PARTICLES} := \sum_i \vec{r}_i \times \vec{p}_i$$

$$L_{PARTICLE} = mvr_{\perp} = (r \sin \theta)p$$

$$\vec{L}_{RIGID} = I_{ABOUT FIXED SKEWER} \vec{\omega}$$

$$\vec{L}_{RIGID} = \vec{L}_{C.O.M. ORBITS ORIGIN} + \vec{L}_{SPIN ABOUT C.O.M.}$$



$$\frac{d\vec{L}}{dt} = \Sigma \vec{\tau}$$

$$\Sigma \vec{L}_i + \int_{t=t_i}^{t=t_f} \left(\sum_{EXT ON SYS} \vec{\tau} \right) dt = \Sigma \vec{L}_f$$

Summing angular momenta

1. Illustrate **before** and **after** situations.
2. Draw **axis of rotation**.
3. Draw positive **sense of rotation**.
4. Determine **sign** of each object's L by determining whether rotation is ccw or cw.

Energy

$$KE_{PARTICLES} := \sum_i \frac{1}{2} \Delta M_i v_i^2$$

$$KE_{CONTINUOUS MASS DISTR.} := \int \frac{1}{2} v^2 dM$$

$$KE_{RIGID} = \frac{1}{2} I_{ABOUT FIXED AXIS} \omega^2 \quad \Delta W_{TF} = \int_{\theta=\theta_i}^{\theta=\theta_f} \tau_F d\theta$$

$$KE_{RIGID} = \frac{1}{2} M v_{C.O.M.}^2 + \frac{1}{2} I_{ABOUT C.O.M.} \omega^2$$