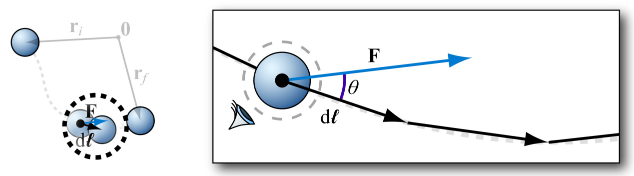
Spatial integration (for AP Physics C Mechanics)

Vector line integral	Mass density	Integrals over distributions

Neatly and graphically represent situation(s)

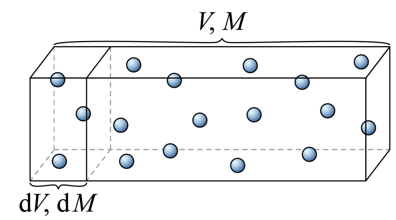
Vector line integral

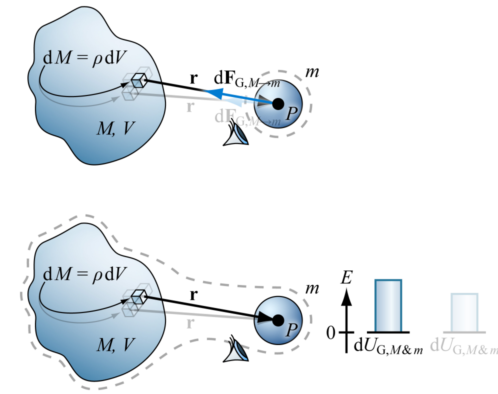
1. Draw a large diagram of the path along which the integral is to be computed.



Mass density / Integrals over distributions

1. Draw a large diagram of the distribution to be integrated over.
2. Draw a reference position, axis of rotation, and/or point of observation P.





Graphically represent quantities and their relationships

2./3. Check whether the physical system exhibits symmetry(ies) permitting use of simplifying coordinate system(s).

Vector line integral

3. If needed, draw a signed coordinate system with a clear origin.
4. Draw a differential displacement $\mathrm{d}\vec{\boldsymbol{\ell}}$. Label this differential displacement with an expression for its length $\mathrm{d}\ell$ (possibly in terms of a coordinate system).
5. Draw the vector $\vec{\mathbf{F}}$ originating from the tail of the displacement vector.
6. Label the vector $\vec{\mathbf{F}}$ with an expression for its magnitude $|\vec{\mathbf{F}}|$ (possibly in terms of a coordinate system).
7. Draw the angle θ between the vector $\vec{\mathbf{F}}$ and $\mathrm{d}\vec{\boldsymbol{\ell}}$.

Mass density / Integrals over distributions

4. If needed, draw a signed coordinate system with a clear origin.
5. Draw a differential element $\mathrm{d}X$ of the distribution, labeled in terms of a density coefficient expression and a differential geometric element (examples include $\mathrm{d}M = \lambda\,\mathrm{d}\ell$, $\mathrm{d}M = \sigma\,\mathrm{d}A$, $\mathrm{d}M = \rho\,\mathrm{d}V$, etc).
6. Draw and label the displacement vector $\vec{\mathbf{r}}$ from the reference position or axis of rotation to the differential element $\mathrm{d}X$ and/or from the differential element $\mathrm{d}X$ to the point of observation P.

Identify relevant allowed starting point (in)equation(s)

Vector line integral

$$Y = \int_{\vec{\mathbf{r}}=\vec{\mathbf{r}}_i}^{\vec{\mathbf{r}}=\vec{\mathbf{r}}_f} \vec{\mathbf{F}}\cdot \mathrm{d}\vec{\boldsymbol{\ell}} = \int_{\vec{\mathbf{r}}=\vec{\mathbf{r}}_i}^{\vec{\mathbf{r}}=\vec{\mathbf{r}}_f} \left(|\vec{\mathbf{F}}|\cos\theta\right)\mathrm{d}\ell$$

8. Use geometry to find a formula or numerical value for the measure of the angle θ.

Mass density

$$\mathrm{d}M = \rho\,\mathrm{d}V$$

For a uniform mass density, $\frac{\mathrm{d}M}{M} = \frac{\mathrm{d}V}{V}$.

Integrals over distributions

$$Y = \int_X f(\vec{\mathbf{r}})\,\mathrm{d}X$$

- Y – output at observation point
- $f(\mathbf{r})$ – expression that depends on spatial relationship between differential element of distribution and observation point
- $\mathrm{d}X$ – differential portion of distribution

7. Use geometry to find a formula or numerical value for the length r of the displacement $\vec{\mathbf{r}}$.

Use numbered steps to show REASoNing

Communicate