Spatial integration (for AP Physics C Mechanics)

Always check whether the physical system exhibits symmetries that permit use of simplifying coordinate systems.

Vector line integral

$$Y = \int \vec{\mathbf{F}} \cdot \mathrm{d}\vec{\boldsymbol{\ell}}$$

$$Y = \int \left(\left| \vec{\mathbf{F}} \right| \cos\theta \right) \mathrm{d}\ell$$

1. Draw a large diagram of the path along which the integral is to be computed.
2. If needed, draw a signed coordinate system with a clear origin.
3. Draw a differential displacement $\mathrm{d}\vec{\boldsymbol{\ell}}$. Label this differential displacement with an expression for its length $\mathrm{d}\ell$ (possibly in terms of a coordinate system).
4. Draw the vector $\vec{\mathbf{F}}$ originating from the tail of the displacement vector.
5. Label the vector $\vec{\mathbf{F}}$ with an expression for its magnitude $\left| \vec{\mathbf{F}} \right|$ (possibly in terms of a coordinate system).
6. Draw the angle θ between the vector $\vec{\mathbf{F}}$ and $\mathrm{d}\vec{\boldsymbol{\ell}}$. Using geometry, find a symbolic formula or numerical value for the measure of this angle.

Integrals over distributions

$$Y = \int f(\vec{\mathbf{r}}) \, \mathrm{d}X$$

Y – output at observation point

$f(\mathbf{r})$ – complicated expression that depends on spatial relationship between differential element of distribution and axis of rotation/point of observation

$\mathrm{d}X$ – differential portion of distribution

1. Draw a large diagram of the distribution to be integrated over.
2. Draw the axis of rotation ℓ or point of observation P.
3. If needed, draw a signed coordinate system with a clear origin.
4. Draw and a differential element $\mathrm{d}X$ of the distribution.
5. Label the differential element with an expression for $\mathrm{d}X$ in terms of a density coefficient expression and a differential geometric element (possibly in terms of a coordinate system). Examples include $\mathrm{d}M = \lambda \, \mathrm{d}x$, $\mathrm{d}M = \sigma \, \mathrm{d}A$, $\mathrm{d}M = \rho \, \mathrm{d}Vol$, etc.
6. Draw and label the displacement vector $\vec{\mathbf{r}}$ from the axis of rotation ℓ to the differential element $\mathrm{d}X$ or from the differential element $\mathrm{d}X$ to the point of observation P.
7. Using geometry, find a symbolic formula or numerical value for the length r of this displacement.