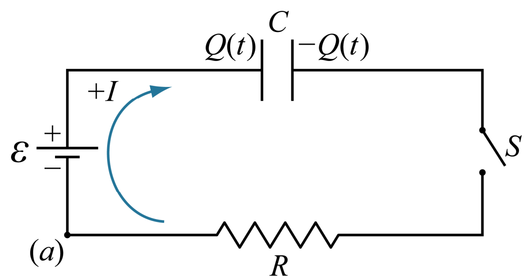
Capacitor charge vs. time in an RC series circuit



$$\Delta V_{\text{LOOP}} = 0$$

$$+\mathcal{E} - \frac{Q}{C} - IR = 0$$

Early times
(just after closing "S")

$$+\mathcal{E} - \frac{\overbrace{Q}^{0}}{C} - IR = 0$$

Capacitor:
$Q_0 = 0,\ \Delta V_0 = 0$

Resistor:
$I_0 = \frac{\mathcal{E}}{R},\ \Delta V_0 = \mathcal{E}$

Intermediate time t

$$+\mathcal{E} - \frac{Q}{C} - \underbrace{I}_{\frac{\mathrm{d}Q}{\mathrm{d}t}} R = 0$$

$$R\frac{\mathrm{d}Q}{\mathrm{d}t} + \frac{1}{C}Q - \mathcal{E} = 0$$

$$\underbrace{RC}_{\tau}\frac{\mathrm{d}Q}{\mathrm{d}t} + Q - C\mathcal{E} = 0$$

$$\tau \int_{Q=Q_i}^{Q=Q_f} \frac{1}{C\mathcal{E} - Q}\,\mathrm{d}Q = \int_{t=t_i}^{t=t_f} \mathrm{d}t$$

$$-\tau\left[\ln|C\mathcal{E} - Q|\right]_{Q=Q_i}^{Q=Q_f} = t_f - t_i$$

Note: $C\mathcal{E} - Q \geq 0$

$$-\tau \ln\left(\frac{C\mathcal{E} - Q_f}{C\mathcal{E} - Q_i}\right) = t_f - t_i$$

	Time	Charge
Initial	$t_i = 0$	$Q_i = 0$
Final	$t_f = t$	$Q_f = Q$

$$\boxed{Q = \underbrace{C\mathcal{E}}_{Q_\infty}\left[1 - e^{-\frac{t}{\tau}}\right]}$$

Late times
(long after closing "S")

$$+\mathcal{E} - \frac{Q}{C} - \underbrace{I}_{0} R = 0$$

Capacitor:
$Q_\infty = C\mathcal{E},\ \Delta V_\infty = \mathcal{E}$

Resistor:
$I_\infty = 0,\ \Delta V_\infty = 0$