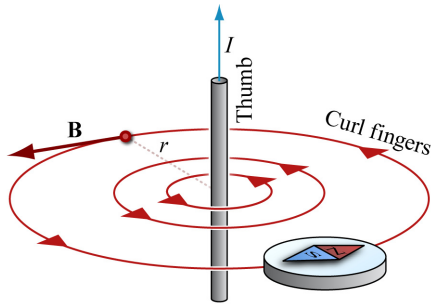


Magnetism

Moving charges create magnetic fields, and magnetic fields exert magnetic forces on moving charges.

Moving charges create magnetic fields

Steady infinite line current





$$|\vec{B}| = \frac{\mu_0 I}{2\pi r}$$

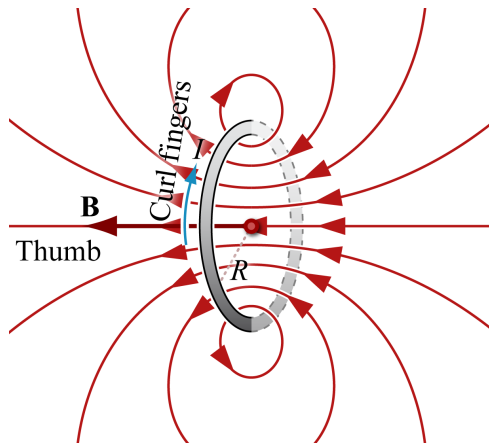
$$[B] = \text{T}$$

$$\mu_0 = 4\pi \times 10^{-7} \frac{\text{T} \cdot \text{m}}{\text{A}}$$

(All panels illustrate **right hand rules**)

Legend:  out (arrowhead)  in (fletching)

Steady circular loop current



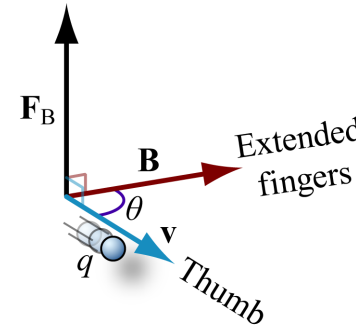
$$|\vec{B}_{\text{CENTER}}| = \frac{\mu_0 I}{2R}$$

Superposition: $\vec{B}_{@P} = \vec{B}_{@P \text{ from src 1}} + \vec{B}_{@P \text{ from src 2}} + \dots$

Magnetic fields exert magnetic forces on moving charges

Force on a moving charge

Out from open palm



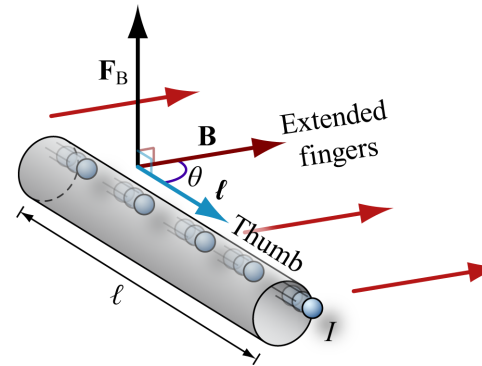
$$\vec{F}_B = q\vec{v} \times \vec{B}$$

(**RHR** for positive moving charge illustrated; for negative moving charge, reverse direction of thumb)

$$|\vec{F}_B| = |q||\vec{v}| \sin \theta |\vec{B}|$$

Force on a line segment of current

Out from open palm



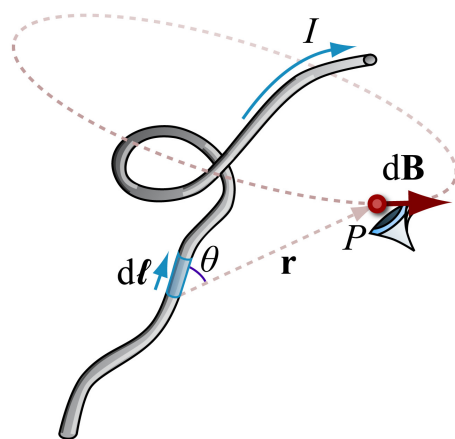
$$\vec{F}_B = I\vec{\ell} \times \vec{B}$$

$$|\vec{F}_B| = I|\vec{\ell}| \sin \theta |\vec{B}|$$

Magnetism with calculus

Calculus can be used to study magnetic fields created by and magnetic forces exerted on more general current distributions.

Biot-Savart Law – for magnetic fields created by steady currents



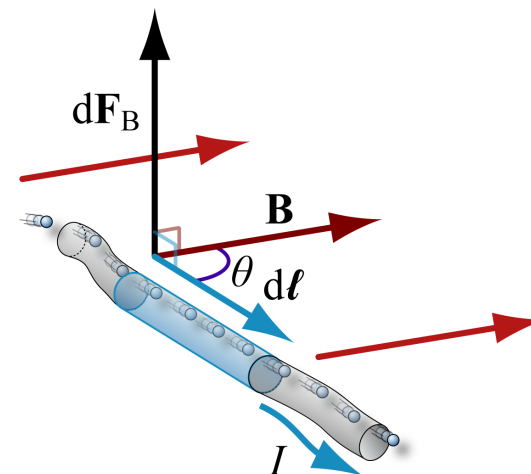
$$d\vec{B} = \frac{\mu_0 I}{4\pi} \frac{d\vec{\ell} \times \hat{r}}{r^2} = \frac{\mu_0 I}{4\pi} \frac{|d\vec{\ell}| \sin \theta}{r^2} \vec{u}_{d\vec{\ell} \times \hat{r}}$$

Superposition

$$\vec{B} = \int d\vec{B}$$

1. Draw current distribution.
2. Draw macaroni $d\vec{\ell}$.
3. Draw observation point P .
4. Draw \vec{r} from macaroni to P .
5. Draw angle θ between macaroni $d\vec{\ell}$ and \vec{r} .
6. Use RHR to find direction of cross product. Draw $d\vec{B}$ in this direction.

Force on a differential current element



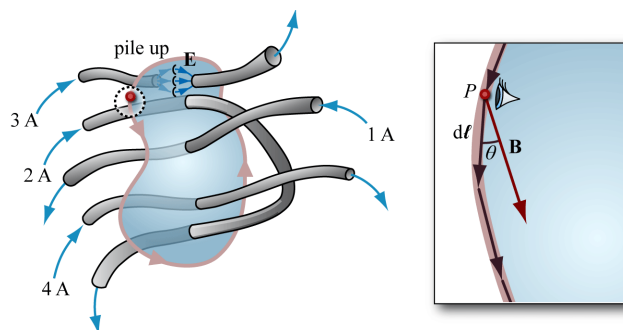
$$d\vec{F}_B = I d\vec{\ell} \times \vec{B} \\ = I |d\vec{\ell}| |\vec{B}| \sin \theta \vec{u}_{d\vec{\ell} \times \vec{B}}$$

1. Draw background \vec{B} -field.
2. Draw current distribution.
3. Draw macaroni $d\vec{\ell}$, local \vec{B} , and angle θ between $d\vec{\ell}$ and local \vec{B} .
4. Use RHR to find direction of magnetic force on current element. Draw $d\vec{F}_B$ in this direction.

Ampère's law

$$\oint \vec{B} \cdot d\vec{\ell} = \mu_0 I_{TOT}$$

$$I_{TOT} = I_{THREAD} + \underbrace{\epsilon_0 \frac{d}{dt} \int \vec{E} \cdot d\vec{A}}_{I_{DISPL}}$$



1. Draw any current(s) and current density(ies).
2. Draw observation point P .
3. Check for useful symmetry.
4. Draw directed encircling loop passing through P .
5. Use RHR for \vec{B} from line currents to determine sign for each current/current density.
6. Draw representative differential step $d\vec{\ell}$ and \vec{B} anchored at point P .
7. Draw angle θ between $d\vec{\ell}$ and \vec{B} .
8. Use angle θ to simplify $\vec{B} \cdot d\vec{\ell}$.