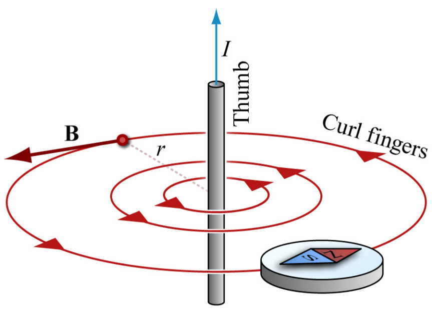
Magnetism

Moving charges create magnetic fields, and magnetic fields exert magnetic forces on moving charges.

Moving charges create magnetic fields

Steady infinite line current



$$\left|\vec{\mathbf{B}}\right| = \frac{\mu_0 I}{2\pi r}$$

$$[\mathrm{B}] = \mathrm{T}$$

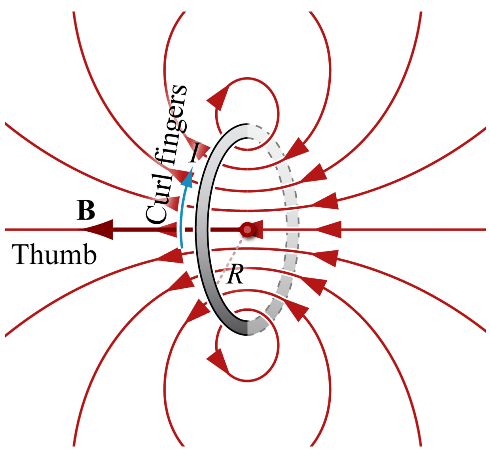
$$\mu_0 = 4\pi \times 10^{-7}\,\frac{\mathrm{T}\cdot\mathrm{m}}{\mathrm{A}}$$

(All panels illustrate **right** hand rules)

Legend: ⊙ out (arrowhead)

⊗ in (fletching)

Steady circular loop current

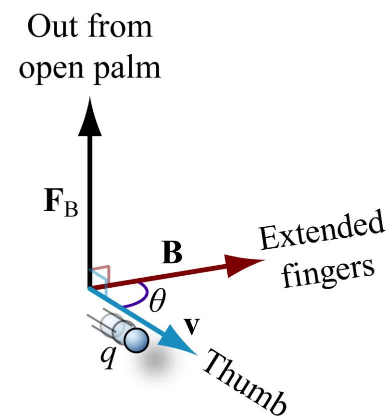


$$\left|\vec{\mathbf{B}}_{\mathrm{CENTER}}\right| = \frac{\mu_0 I}{2R}$$

Superposition: $\vec{\mathbf{B}}_{@P} = \vec{\mathbf{B}}_{@P\text{ from src 1}} + \vec{\mathbf{B}}_{@P\text{ from src 2}} + \cdots$

Magnetic fields exert magnetic forces on moving charges

Force on a moving charge

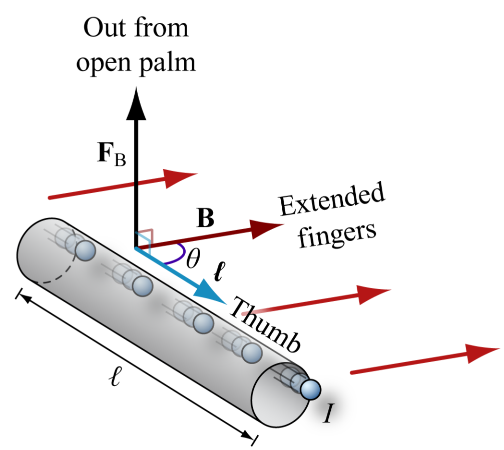


$$\vec{\mathbf{F}}_{\mathrm{B}} = q\vec{\mathbf{v}} \times \vec{\mathbf{B}}$$

(**RHR** for positive moving charge illustrated; for negative moving charge, reverse direction of thumb)

$$\left|\vec{\mathbf{F}}_{\mathrm{B}}\right| = |q|\,|\vec{\mathbf{v}}|\sin\theta\,\left|\vec{\mathbf{B}}\right|$$

Force on a line segment of current



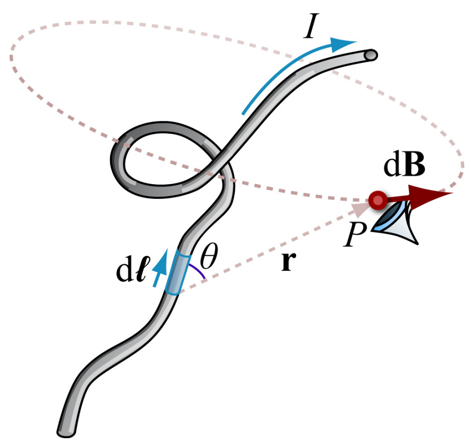
$$\vec{\mathbf{F}}_{\mathrm{B}} = I\vec{\boldsymbol{\ell}} \times \vec{\mathbf{B}}$$

$$\left|\vec{\mathbf{F}}_{\mathrm{B}}\right| = I\left|\vec{\boldsymbol{\ell}}\right|\sin\theta\,\left|\vec{\mathbf{B}}\right|$$

Magnetism with calculus

Calculus can be used to study magnetic fields created by and magnetic forces exerted on more general current distributions.

Biot-Savart Law – for magnetic fields created by steady currents



$$\mathrm{d}\vec{\mathbf{B}} = \frac{\mu_0}{4\pi}\frac{I\,\mathrm{d}\vec{\boldsymbol{\ell}} \times \hat{\mathbf{r}}}{r^2} = \frac{\mu_0}{4\pi}\frac{I\left|\mathrm{d}\vec{\boldsymbol{\ell}}\right|\sin\theta}{r^2}\vec{\mathbf{u}}_{\mathrm{d}\vec{\mathbf{B}}}$$

Superposition

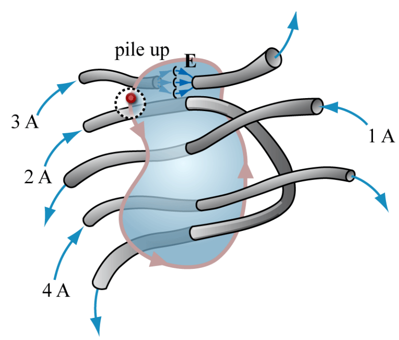
$$\vec{\mathbf{B}} = \int \mathrm{d}\vec{\mathbf{B}}$$

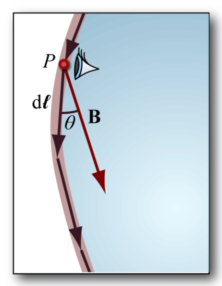
1. Draw current distribution.
2. Draw macaroni $\mathrm{d}\vec{\boldsymbol{\ell}}$.
3. Draw observation point P.
4. Draw $\vec{\mathbf{r}}$ from macaroni to P.
5. Draw angle θ between macaroni $\mathrm{d}\vec{\boldsymbol{\ell}}$ and $\vec{\mathbf{r}}$.
6. Use RHR to find direction of cross product. Draw $\mathrm{d}\vec{\mathbf{B}}$ in this direction.

Ampère's law

$$\oint \vec{\mathbf{B}} \cdot \mathrm{d}\vec{\boldsymbol{\ell}} = \mu_0 I_{\mathrm{TOT}}$$

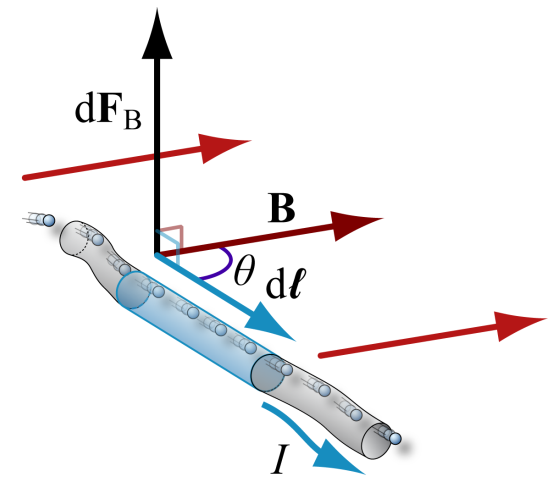
$$I_{\mathrm{TOT}} = I_{\mathrm{THREAD}} + \underbrace{\epsilon_0 \frac{\mathrm{d}}{\mathrm{d}t}\int \vec{\mathbf{E}} \cdot \mathrm{d}\vec{\mathbf{A}}}_{I_{\mathrm{DISPL}}}$$





1. Draw any current(s) and current density(ies).
2. Draw observation point P.
3. Check for useful symmetry.
4. Draw directed encircling loop passing through P.
5. Use RHR for $\vec{\mathbf{B}}$ from line currents to determine sign for each current/current density.
6. Draw representative differential step $\mathrm{d}\vec{\boldsymbol{\ell}}$ and $\vec{\mathbf{B}}$ anchored at point P.
7. Draw angle θ between $\mathrm{d}\vec{\boldsymbol{\ell}}$ and $\vec{\mathbf{B}}$.
8. Use angle θ to simplify $\vec{\mathbf{B}} \cdot \mathrm{d}\vec{\boldsymbol{\ell}}$.

Force on a differential current element



$$\begin{aligned}\mathrm{d}\vec{\mathbf{F}}_{\mathrm{B}} &= I\,\mathrm{d}\vec{\boldsymbol{\ell}} \times \vec{\mathbf{B}} \\ &= I\left|\mathrm{d}\vec{\boldsymbol{\ell}}\right|\left|\vec{\mathbf{B}}\right|\sin\theta\,\vec{\mathbf{u}}_{\mathrm{d}\vec{\mathbf{F}}_{\mathrm{B}}}\end{aligned}$$

1. Draw background $\vec{\mathbf{B}}$-field.
2. Draw current distribution.
3. Draw macaroni $\mathrm{d}\vec{\boldsymbol{\ell}}$, local $\vec{\mathbf{B}}$, and angle θ between $\mathrm{d}\vec{\boldsymbol{\ell}}$ and local $\vec{\mathbf{B}}$.
4. Use RHR to find direction of magnetic force on current element. Draw $\mathrm{d}\vec{\mathbf{F}}_{\mathrm{B}}$ in this direction.