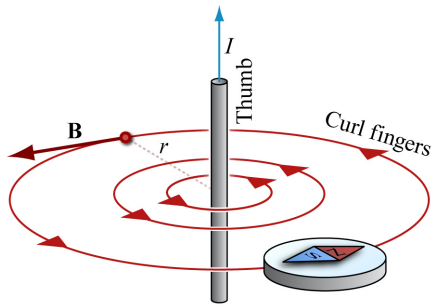


# Magnetism

Moving charges create magnetic fields, and magnetic fields exert magnetic forces on moving charges.

Moving charges create magnetic fields

## Infinite line current





$$|\vec{B}| = \frac{\mu_0 I}{2\pi r}$$

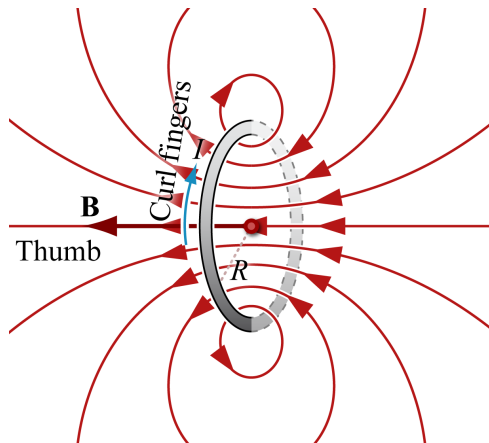
$$[B] = \text{T}$$

$$\mu_0 = 4\pi \times 10^{-7} \frac{\text{T} \cdot \text{m}}{\text{A}}$$

(All panels illustrate right hand rules)

Legend:  out (arrowhead)   
  in (fletching)

## Circular loop current



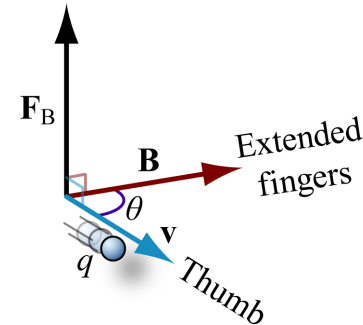
$$|\vec{B}_{\text{CENTER}}| = \frac{\mu_0 I}{2R}$$

Superposition:  $\vec{B}_{@P} = \vec{B}_{@P \text{ from src 1}} + \vec{B}_{@P \text{ from src 2}} + \dots$

Magnetic fields exert magnetic forces on moving charges

## Force on a moving charge

Out from open palm



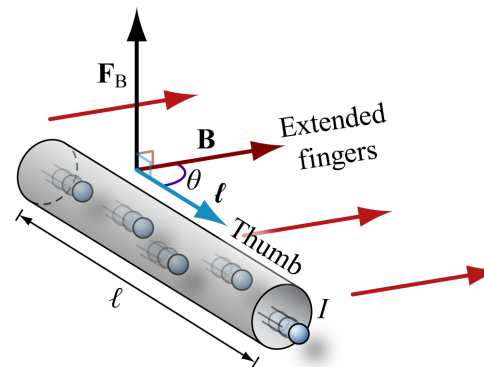
$$\vec{F}_B = q\vec{v} \times \vec{B}$$

(RHR for positive moving charge illustrated; for negative moving charge, reverse direction of thumb)

$$|\vec{F}_B| = |q| |\vec{v}| \sin \theta |\vec{B}|$$

## Force on a line segment of current

Out from open palm



$$\vec{F}_B = I\vec{\ell} \times \vec{B}$$

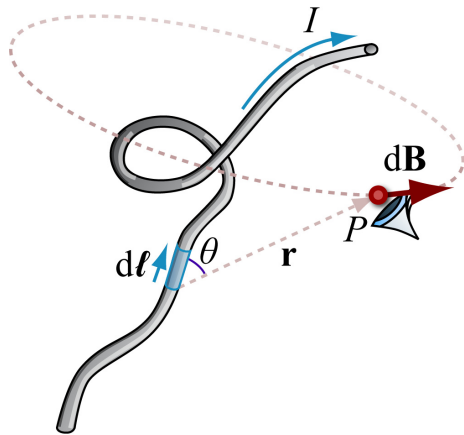
$$|\vec{F}_B| = I |\vec{\ell}| \sin \theta |\vec{B}|$$

## Magnetism with calculus

# Magnetism with calculus

Calculus can be used to study magnetic fields created by and magnetic forces exerted on more general current distributions.

## Biot-Savart Law



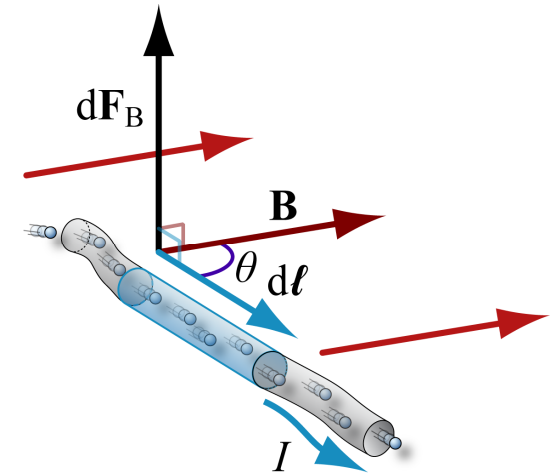
$$d\vec{B} = \frac{\mu_0 I d\vec{\ell} \times \hat{r}}{4\pi r^2} = \frac{\mu_0 I |d\vec{\ell}| \sin \theta}{4\pi r^2} \vec{u}_{d\vec{B}}$$

Superposition

$$\vec{B} = \int d\vec{B}$$

1. Draw current distribution.
2. Draw macaroni  $d\vec{\ell}$ .
3. Draw observation point  $P$ .
4. Draw  $\vec{r}$  from macaroni to  $P$ .
5. Draw angle  $\theta$  between macaroni  $d\vec{\ell}$  and  $\vec{r}$ .
6. Use RHR to find direction of cross product. Draw  $d\vec{B}$  in this direction.

## Force on a differential current element



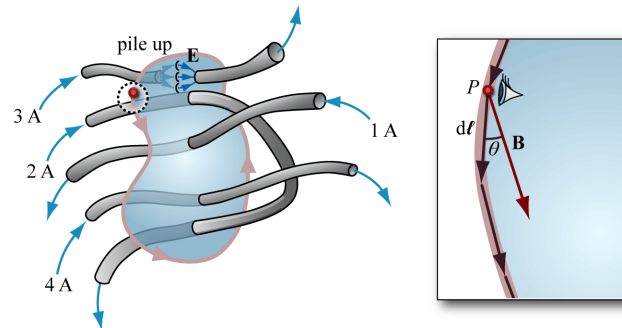
$$d\vec{F}_B = I d\vec{\ell} \times \vec{B} \\ = I |d\vec{\ell}| |\vec{B}| \sin \theta \vec{u}_{d\vec{F}_B}$$

1. Draw background  $\vec{B}$ -field.
2. Draw current distribution.
3. Draw macaroni  $d\vec{\ell}$ , local  $\vec{B}$ , and angle  $\theta$  between  $d\vec{\ell}$  and local  $\vec{B}$ .
4. Use RHR to find direction of magnetic force on current element. Draw  $d\vec{F}_B$  in this direction.

## Ampère's law

$$\oint \vec{B} \cdot d\vec{\ell} = \mu_0 I_{TOT}$$

$$I_{TOT} = I_{THREAD} + \underbrace{\epsilon_0 \frac{d}{dt} \int \vec{E} \cdot d\vec{A}}_{I_{DISPL}}$$



1. Draw any current(s) and current density(ies).
2. Draw observation point  $P$ .
3. Check for useful symmetry.
4. Draw directed encircling loop passing through  $P$ .
5. Use RHR for  $\vec{B}$  from line currents to determine sign for each current/current density.
6. Draw representative differential step  $d\vec{\ell}$  and  $\vec{B}$  anchored at point  $P$ .
7. Draw angle  $\theta$  between  $d\vec{\ell}$  and  $\vec{B}$ .
8. Use angle  $\theta$  to simplify  $\vec{B} \cdot d\vec{\ell}$ .