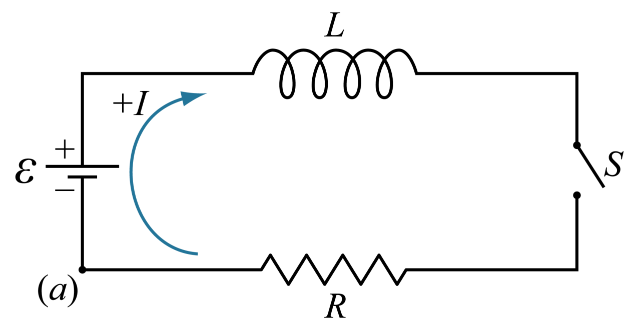
Increasing current over time in an LR series circuit



$$\Delta V_{\text{LOOP}} = 0$$

$$+\mathcal{E} - L\frac{\mathrm{d}I}{\mathrm{d}t} - IR = 0$$

Early times
(just after closing "S")

$$+\mathcal{E} - L\frac{\mathrm{d}I}{\mathrm{d}t} - \underbrace{I}_{0}R = 0$$

Inductor:
$I_0 = 0,\ \Delta V_0 = \mathcal{E}$

Resistor:
$I_0 = 0,\ \Delta V_0 = 0$

Intermediate time t

$$+\mathcal{E} - L\frac{\mathrm{d}I}{\mathrm{d}t} - IR = 0$$

$$-L\frac{\mathrm{d}I}{\mathrm{d}t} = IR - \mathcal{E}$$

$$-\underbrace{\left(\frac{L}{R}\right)}_{\tau}\frac{\mathrm{d}I}{\mathrm{d}t} = I - \underbrace{\left(\frac{\mathcal{E}}{R}\right)}_{I_\infty}$$

$$-\tau\int_{I=I_i}^{I=I_f}\frac{1}{I - I_\infty}\,\mathrm{d}I = \int_{t=t_i}^{t=t_f}\mathrm{d}t$$

$$-\tau\left[\ln|I - I_\infty|\right]_{I=I_i}^{I=I_f} = t_f - t_i$$

Note: $I - I_\infty < 0$

$$-\tau\ln\left(\frac{I_\infty - I_f}{I_\infty - I_i}\right) = t_f - t_i$$

	Time	Current
Initial	$t_i = 0$	$I_i = 0$
Final	$t_f = t$	$I_f = I$

$$\boxed{I = \underbrace{\left(\frac{\mathcal{E}}{R}\right)}_{I_\infty}\left[1 - e^{-\frac{t}{\tau}}\right]}$$

Late times
(long after closing "S")

$$+\mathcal{E} - L\underbrace{\frac{\mathrm{d}I}{\mathrm{d}t}}_{0} - IR = 0$$

Inductor:
$I_\infty = \frac{\mathcal{E}}{R},\ \Delta V_\infty = 0$

Resistor:
$I_\infty = \frac{\mathcal{E}}{R},\ \Delta V_\infty = \mathcal{E}$