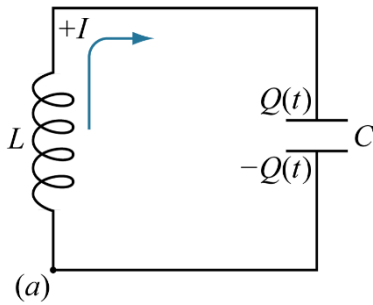


Oscillations in LC and RLC series circuits

LC series circuit: Oscillations



$$\Delta V_{\text{LOOP}} = 0$$

$$\text{cw: } -L \frac{dI}{dt} - \frac{Q}{C} = 0$$

$$I = \frac{dQ}{dt}$$

$$-L \frac{d^2Q}{dt^2} - \frac{Q}{C} = 0$$

$$\frac{d^2Q}{dt^2} = -\frac{1}{LC}Q$$

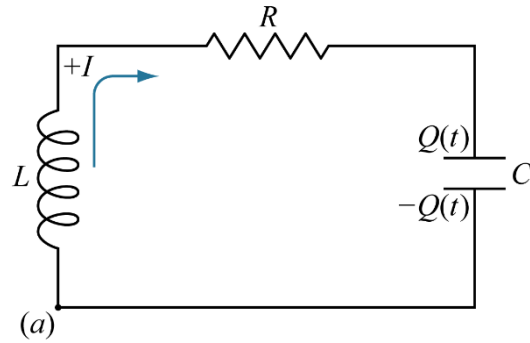
Recall SHM

$$\frac{d^2x}{dt^2} = -\omega^2x \Rightarrow x = x_0 \cos(\omega t + \phi)$$

$$Q(t) = Q_{\text{MAX}} \cos\left(\frac{1}{\sqrt{LC}}t + \phi\right)$$

Q_{MAX} and ϕ are constants.

RLC series circuit: Damped oscillations



$$\Delta V_{\text{LOOP}} = 0$$

$$\text{cw: } -L \frac{dI}{dt} - IR - \frac{Q}{C} = 0$$

$$I = \frac{dQ}{dt}$$

$$\frac{d^2Q}{dt^2} + \frac{R}{L} \frac{dQ}{dt} + \frac{1}{LC}Q = 0$$

Try $Q(t) = e^{\lambda t}$

$$(\lambda^2 e^{\lambda t}) + \frac{R}{L}(\lambda e^{\lambda t}) + \frac{1}{LC}e^{\lambda t} = 0$$

$$\lambda^2 + \frac{R\lambda}{L} + \frac{1}{LC} = 0$$

$$\lambda = \frac{-\frac{R}{L} \pm \sqrt{\left(\frac{R}{L}\right)^2 - 4(1)\left(\frac{1}{LC}\right)}}{2}$$

$$\lambda = -\frac{R}{2L} \pm i \sqrt{\frac{1}{LC} - \left(\frac{R}{2L}\right)^2}$$

Consider two example solutions

$$Q_1(t) = e^{i\phi} e^{-\frac{R}{2L}t + i\sqrt{\frac{1}{LC} - \left(\frac{R}{2L}\right)^2}t}$$

$$Q_2(t) = e^{-i\phi} e^{-\frac{R}{2L}t - i\sqrt{\frac{1}{LC} - \left(\frac{R}{2L}\right)^2}t}$$

Let

$$Q(t) = \frac{Q(0)}{\cos \phi} \left(\frac{Q_1(t) + Q_2(t)}{2} \right)$$

$$Q(t) = \frac{Q(0)}{\cos \phi} e^{-\frac{R}{2L}t} \cos\left(\sqrt{\frac{1}{LC} - \left(\frac{R}{2L}\right)^2}t + \phi\right)$$