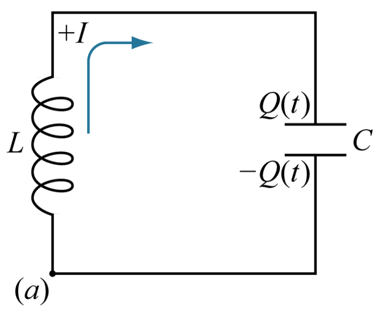
Oscillations in LC and RLC series circuits

LC series circuit: Oscillations



$$\Delta V_{\mathrm{LOOP}} = 0$$

$$\text{cw:} \; -L\frac{\mathrm{d}I}{\mathrm{d}t} - \frac{Q}{C} = 0$$

$$I = \frac{\mathrm{d}Q}{\mathrm{d}t}$$

$$-L\frac{\mathrm{d}^2Q}{\mathrm{d}t^2} - \frac{Q}{C} = 0$$

$$\frac{\mathrm{d}^2Q}{\mathrm{d}t^2} = -\frac{1}{LC}Q$$

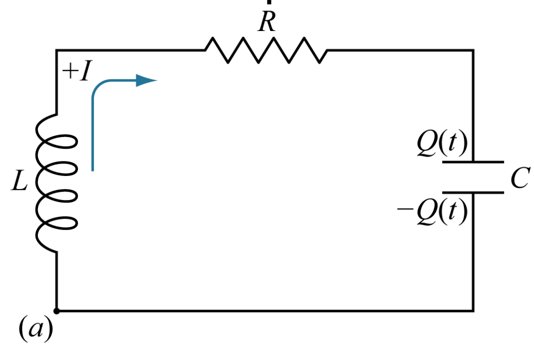
Recall SHM

$$\frac{\mathrm{d}^2x}{\mathrm{d}t^2} = -\omega^2 x \Rightarrow x(t) = A\cos(\omega t + \delta)$$

$$Q(t) = Q_{\mathrm{MAX}}\cos\left(\frac{1}{\sqrt{LC}}t + \delta\right)$$

Q_{MAX} and δ are constants.

RLC series circuit: Damped oscillations



$$\Delta V_{\mathrm{LOOP}} = 0$$

$$\text{cw:} \; -L\frac{\mathrm{d}I}{\mathrm{d}t} - IR - \frac{Q}{C} = 0$$

$$I = \frac{\mathrm{d}Q}{\mathrm{d}t}$$

$$\frac{\mathrm{d}^2Q}{\mathrm{d}t^2} + \frac{R}{L}\frac{\mathrm{d}Q}{\mathrm{d}t} + \frac{1}{LC}Q = 0$$

Try $Q(t) = e^{\lambda t}$

$$\left(\lambda^2 \cancel{e^{\lambda t}}\right) + \frac{R}{L}\left(\lambda \cancel{e^{\lambda t}}\right) + \frac{1}{LC}\cancel{e^{\lambda t}} = 0$$

$$\lambda^2 + \frac{R\lambda}{L} + \frac{1}{LC} = 0$$

$$\lambda = \frac{-\frac{R}{L} \pm \sqrt{\left(\frac{R}{L}\right)^2 - 4(1)\left(\frac{1}{LC}\right)}}{2}$$

$$\lambda = -\frac{R}{2L} \pm i\sqrt{\frac{1}{LC} - \left(\frac{R}{2L}\right)^2}$$

Consider two example solutions

$$Q_1(t) = e^{i\delta} e^{-\frac{R}{2L}t + i\sqrt{\frac{1}{LC} - \left(\frac{R}{2L}\right)^2}\,t}$$

$$Q_2(t) = e^{-i\delta} e^{-\frac{R}{2L}t - i\sqrt{\frac{1}{LC} - \left(\frac{R}{2L}\right)^2}\,t}$$

Let

$$Q(t) = \frac{Q(0)}{\cos\delta}\left(\frac{Q_1(t) + Q_2(t)}{2}\right)$$

$$Q(t) = \frac{Q(0)}{\cos\delta} e^{-\frac{R}{2L}t} \cos\left(\sqrt{\frac{1}{LC} - \left(\frac{R}{2L}\right)^2}\; t + \delta\right)$$