

Connections between arbitrage-free pricing and equilibrium statistical mechanics

If you come from a physics background, the following approximate correspondence chart might be of interest.

	Equilibrium statistical mechanics	Arbitrage-free pricing
Underlying notions	Properties of Hamiltonian systems	Assumptions about human goals
Fundamental principle	Fundamental postulate: Given sufficient time, an isolated Hamiltonian system will eventually explore all accessible states/cells of phase space with equal weight	No-arbitrage assumption: The only prices that can trade with enduring abundance are those prices that cannot be used to build an arbitrage portfolio.
Extension of fundamental principle to method for computing observable quantities using distributions	Apply Law Of The Unconscious Statistician (LOTUS) to obtain expected values of physical observables at thermal equilibrium	Fundamental theorem of asset pricing (FTAP): Imposing no-arbitrage assumption is equivalent to computing present asset price ratios in terms of weighted combinations of possible future price ratios
Storing and using distributional information	Partition function can be used, with appropriate manipulations involving partial derivatives, to obtain expected values of physical observables at thermal equilibrium	Risk-neutral distribution can be encoded in call option prices (option price-probability duality) and implied volatilities.
Commonly used assumptions about scale	Thermodynamic limit: System is much larger than characteristic scale of spatial correlations among within-system components, so physical quantities are either intensive or extensive (proportional to system size).	Market is deep so that number of units of asset extracted from or transferred to market does not affect the unit price of the asset. The price of a number of units of an asset is proportional to the number of units of the asset.
Approximate universality and distributional assumptions	<p>Suppose the system of interest is much smaller than surroundings. Consider a characteristic unit of energy leaving the system and entering the surroundings. The energy unit encounters a set of possible places/ways to dwell in the parts of the surroundings. The surroundings are so large (made of so many parts) that the introduction of the energy unit hardly fills up the surroundings. The surroundings almost seem as empty as before the energy unit was introduced in the sense that a second characteristic energy unit introduced into the surroundings encounters practically the same number of possible places/ways to dwell in the parts of the surroundings. Thus, the factor by which the number of microstates of the surroundings increases with each unit of energy transferred from the system to the surroundings is constant.</p> <p>The conventional exponential expression used to keep track of the factor by which the number of microstates of the surroundings changes as energy is transferred to the surroundings is the Boltzmann factor.</p>	<p>Suppose an asset price can change through a time-sequence of independentish and similarly-sized fold changes. As an approximation, say that the asset price can change through a time-sequence of i.i.d. fold changes on a binomial tree. The risk-neutral distribution for the asset price is also obtained through a time-sequence of i.i.d. fold changes on the same binomial tree (but not necessarily using "actual" probabilities). The resulting risk-neutral distribution for the eventual (later time) asset price is thus log-normal.</p> <p>Price formulas obtained under such assumptions (e.g. Black-Scholes formula) are called Black formulas.</p>