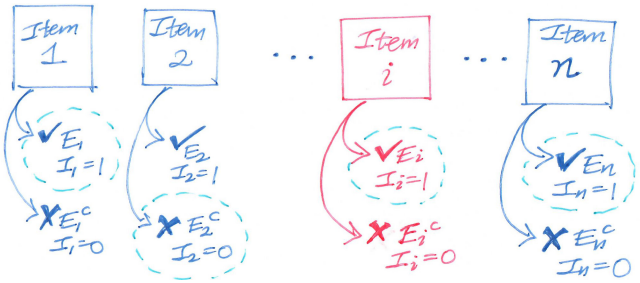
BASIC PROBABILITY

Ontological shifting

Thinking of an event in terms of a specification of the states of a collection of n yes/no checklist items

"In order to …, the … would need to …, the … would need to …, and the … would need to …."



Big ideas

Tracking overlap

Inclusion-exclusion

$$P\left(\bigcup_{i=1}^{n} E_i\right) = \sum_{i=1}^{n} P(E_i) + \Delta P_{\cap}$$

$$\Delta P_{\cap} = -\sum_{i<j} P(E_i \cap E_j) + \sum_{i<j<k} P(E_i \cap E_j \cap E_k) - \cdots + (-1)^{n-1} \sum_{i<j<\cdots<\ell_{\text{LAST}}} P(E_i \cap E_j \cap \cdots \cap E_{\ell_{\text{LAST}}})$$

Connect indicators to probability

If

- I_j is the indicator for the event that the jth coin-toss turns up heads
- $X =$ number of heads among n coin tosses

then

$$X = I_1 + I_2 + \cdots + I_n$$

Fundamental bridge

$$P(A) = \mathbb{E}[I_A]$$

Details

Probability

Definition of probability

$$P(A) := \frac{\text{number of outcomes in event } A}{\text{number of outcomes overall}}$$

(Rearranged) definition of conditional probability

$$P(A \cap B) = P(A|B)P(B)$$

Law of total probability (LOTP)

$$P(A) = \sum_i P(A|E_i)P(E_i)$$

Indicators

Definition of indicator

$$I_A := \begin{cases} 1, & A \text{ happened} \\ 0, & A \text{ didn't happen} \end{cases}$$