

啰嗦 ways to do math and science

Problem types	Activity category	Scaffold	Pg.	Related topics	References
Test experiment	Inquire (reject candidate models)	STEAM	1	inquiry-based learning	Etkina <i>et al.</i> https://www.islephysics.net/
				Modeling Instruction™ patternicity	AMTA https://www.modelinginstruction.org/ Shermer 2008 doi.org/10.1038/scientificamerican1208-48
Word problems	Read & write notes & questions	SCAN	2	multiple representations	Larkin, McDermott, Simon, and Simon 1980 doi.org/10.1126/science.208.4450.1335 van Heuvelen 1991 doi.org/10.1119/1.16668 Liao 2018 doi.org/10.1119/1.5028250
				idea distinct from name	Arons 1984 doi.org/10.1119/1.2341444
Algebra problems Geometry proofs Physics & chemistry problems	Argue formally	STEP	3	Formal arguments	Toulmin 2003 doi.org/10.1017/CBO9780511840005 McNeill, Lizotte, Krajcik, and Marx 2006 doi.org/10.1207/s15327809jls1502_1
				chains of reasoning	Speirs, Ferm, Stetzer, and Lindsey 2016 doi.org/10.1119/perc.2016.pr.077
Physics and chemistry problems	Spider-web thinking (track quantities and governing and constitutive relationships that simultaneously constrain them)	MAP	4	Novice and expert problem-solving, production schemas	Chi, Feltovich, and Glaser 1981 doi.org/10.1207/s15516709cog0502_2
				Simultaneously diagramming governing and constitutive relationships	Rosengrant 2011 doi.org/10.1119/1.3527754

Carry out **inquiry** (reject candidate models) using **STEAM**.

Example: What is $\sqrt{x^2}$? Alice proposes that $\sqrt{x^2} = x$, and Beth propose that $\sqrt{x^2} = |x|$.

	Shiftable input(s)	Theoretical Expectations		Actual results and Measurements
	x	By Alice's model $\sqrt{x^2} = x$,	By Beth's model $\sqrt{x^2} = x $,	$\sqrt{x^2}$
1.	-2	-2 😞	2	$\sqrt{(-2)^2} = \sqrt{4} = 2$
2.	-1	-1 😞	1	$\sqrt{(-1)^2} = \sqrt{1} = 1$
3.	0	0	0	$\sqrt{(0)^2} = \sqrt{0} = 0$
4.	1	1	1	$\sqrt{(1)^2} = \sqrt{1} = 1$
5.	2	2	2	$\sqrt{(2)^2} = \sqrt{4} = 2$

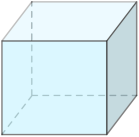
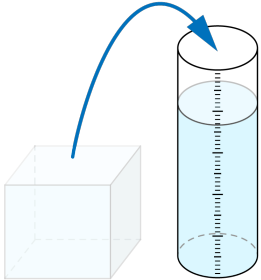
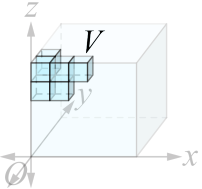
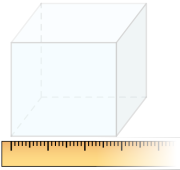
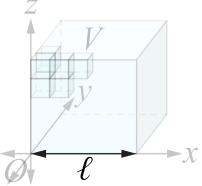
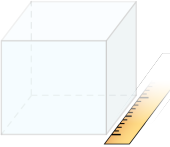
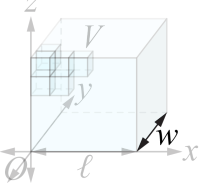
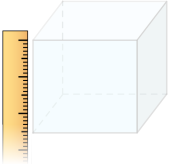
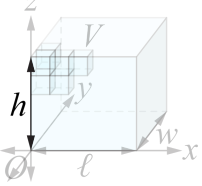
In the STEAM table, some of Alice's predictions are **inconsistent** with actual values (when x is negative), so **reject** Alice's model, $\sqrt{x^2} = x$.

In the STEAM table, all of Beth's predictions are **consistent** with actual values, so Beth's model $\sqrt{x^2} = |x|$ **remains in play**. The STEAM table alone doesn't allow Beth's model to be "ruled in" since the STEAM table doesn't preclude the possibility of additional values of the shiftable input x resulting in values of $\sqrt{x^2}$ that turn out to fail to equal $|x|$.

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Read and write notes and questions using SCAN.

Example: Translate the sentence, “A rectangular parcel of water has a volume equaling the parcel’s length times the parcel’s width times the parcel’s height.”

	Short phrase	Cartoon	Axes and plots	Notation
1.	A rectangular parcel of water	Semi-transparent rectangular prism 		
2.	has a volume	Arrow indicating water can be poured from rectangular container into graduated cylinder used to measure volume 	Cartesian axis system implying scale for representative unit cubes within parcel 	Let $V = \text{Volume}$
3.	equaling		Optional: Number line with two arrows pointing from the origin to same tickmark	$V =$
4.	the parcel’s length	Optional: Ruler used to measure length 	Double-headed arrow indicating size of an edge parallel to x -axis 	Let $\ell = \text{width}$ $V = \ell$
5.	times		Optional: Area model of multiplication	$V = \ell \cdot$
6.	the parcel’s width	Optional: Ruler used to measure width 	Double-headed arrow indicating size of an edge parallel to y -axis 	Let $w = \text{width}$ $V = \ell \cdot w$
7.	times		Optional: Area model of multiplication	$V = \ell \cdot w \cdot$
8.	the parcel’s height.	Optional: Ruler used to measure height 	Double-headed arrow indicating size of an edge parallel to z -axis 	Let $h = \text{height}$ $V = \ell \cdot w \cdot h$

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Argue formally using STEP.

Example: Solve for x . Show each step that can't be accomplished on a basic four-function calculator.

$$4x - 2 = 98$$

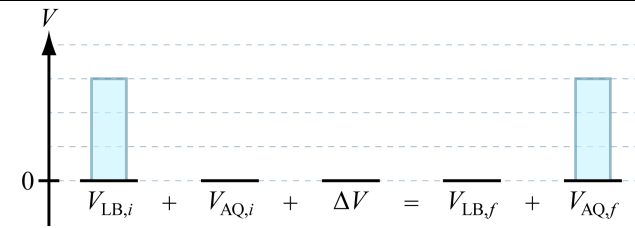
	Statement	Tool	Equivalent parts (of statement & tool)	Populated tool
1.	$4x - 2 = 98$	$a = b$ \Downarrow $a + c = b + c$	$a = 4x - 2$ $b = 98$ $c = 2$	$4x - 2 = 98$ \Downarrow $4x - 2 + 2 = 98 + 2$
2.	$4x + 0 = 100$	$a \leftrightarrow a + 0 \leftrightarrow 0 + a$	$a = 4x$	$4x \leftrightarrow 4x + 0 \leftrightarrow 0 + 4x$
3.	$4x = 100$	$a = b$ \Downarrow $\frac{a}{c} = \frac{b}{c}, c \neq 0$	$a = 4x$ $b = 100$ $c = 4$	$4x = 100$ \Downarrow $\frac{4x}{4} = \frac{100}{4}, 4 \neq 0$
4.	$\frac{4x}{4} = 25$	$a \leftrightarrow a \cdot 1 \leftrightarrow 1 \cdot a$	$a = 4$	$4 \leftrightarrow 4 \cdot 1 \leftrightarrow 1 \cdot 4$
5.	$\frac{4x}{4 \cdot 1} = 25$	$\frac{ab}{cd} \leftrightarrow \frac{a b}{c d}$	$a = 4$ $b = x$ $c = 4$ $d = 1$	$\frac{4x}{4 \cdot 1} \leftrightarrow \frac{4 x}{4 1}$
6.	$\frac{4 x}{4 1} = 25$	$\frac{a}{a}, a \neq 0 \leftrightarrow 1$	$a = 4$	$\frac{4}{4}, 4 \neq 0 \leftrightarrow 1$
7.	$1 \cdot \frac{x}{1} = 25$	$\frac{a}{1} \leftrightarrow a$	$a = x$	$\frac{x}{1} \leftrightarrow x$
8.	$1 \cdot x = 25$	$a \leftrightarrow a \cdot 1 \leftrightarrow 1 \cdot a$	$a = x$	$x \leftrightarrow x \cdot 1 \leftrightarrow 1 \cdot x$
9.	$x = 25$			

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Perform **spider-web thinking** (track quantities and governing and constitutive relationships that constrain them) using **MAP**.

Example: Make a MAP for the following problem:

A rectangular lunchbox full of water is emptied completely into an initially empty rectangular aquarium that's longer, wider, and taller than the lunchbox. Is the height of the water in the aquarium greater than, less than, or the same as the height of the water initially in the lunchbox?



	Engineer's jargon		Name	Notation	Preferred layout
Managers	Governing relationships among properties belonging to various objects in the system or to interactions with the environment	1.	Volume conservation for incompressible fluid	$V_{TOT,BEGIN} + \Delta V_{GAINED} - \Delta V_{LOST} = V_{TOT,END}$	Near center/top Horizontal
Associated Personnel	Constitutive relationships among properties belonging to an individual object within a system or to an interaction with the environment	2.	Initial volume of rectangular parcel of water in lunchbox	$V_{LB,i} = \ell_{LB,i} \cdot w_{LB,i} \cdot h_{LB,i}$	Near bottom/edges Vertical
		3.	Initial volume of rectangular parcel of water in aquarium	$V_{AQ,i} = \ell_{AQ,i} \cdot w_{AQ,i} \cdot h_{AQ,i}$	
		4.	Final volume of rectangular parcel of water in lunchbox	$V_{LB,f} = \ell_{LB,f} \cdot w_{LB,f} \cdot h_{LB,f}$	
		5.	Final volume of rectangular parcel of water in aquarium	$V_{AQ,f} = \ell_{AQ,f} \cdot w_{AQ,f} \cdot h_{AQ,f}$	



Read **Manager** relationships horizontally. Read **Associated Personnel** relationships vertically. When investigating a value, consider the possibly multiple relationships that constrain the value, just like when you consider the multiple clues that a single letter must sometimes satisfy in a crossword puzzle.