Survival skills for reading and reasoning

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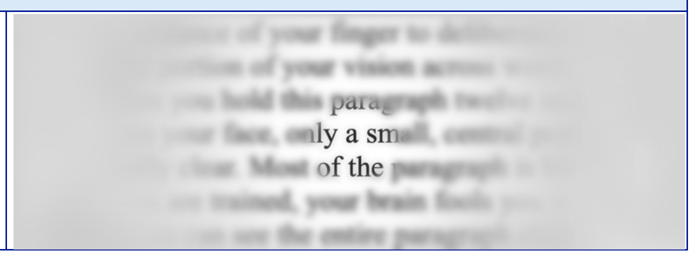
Use this packet to develop skills helpful for basic reading and reasoning useful through Algebra 1. To learn additional methods for geometry and physics, use “Cognitive Steps for Learning, Applying, and Explaining Physics Without Using Talent” https://davidliao.com/handouts/Physics/11.0%20Summer%20Mechanics/Mastering%20Tests%20of%20Physics%20Reasoning.pdf. Throughout this document, “read” means pointing and reading aloud:

Point as you read aloud 指差喚呼	
Use the guidance of your finger to deliberately drag the central portion of your vision across words as you read. When you hold this paragraph twelve inches away from your face, only a small, central portion of it is actually clear. Most of the paragraph is blurry. Unless you are trained, your brain fools you into thinking you can see the entire paragraph clearly.	

Translate text into a language-independent format

Translate the following passage into pictures or tables that are easy for people who don't read English to understand.

Alice stands on a box that has a height of 2 feet. The top of Alice's head is 5 feet above the ground. How tall is Alice?

Reading steps

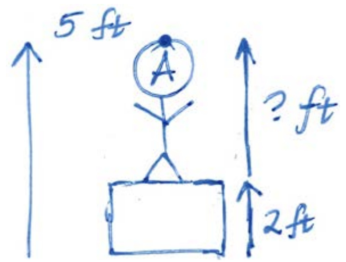
1. Break passage into short phrases, each containing ~1-3 (2 is ideal) of the following: **noun**, **verb**, and **preposition**.
2. **Be sure to know which noun each pronoun stands for**. Unsure for a particular pronoun? Work backward from the pronoun until you find a noun that isn't a pronoun. Read the phrase containing the pronoun, replacing the pronoun with the noun you've found. Does the phrase sound reasonable? If not, go further back to find a noun that isn't a pronoun. Read the phrase containing the pronoun with candidate nouns until you have a sensible phrase.
3. **Be sure to know which noun each preposition helps specify**. Unsure for a particular preposition? Work backward from the preposition until you find a noun that isn't a pronoun. Read the noun immediately followed by the prepositional phrase. Does the phrase sound reasonable? If not, go further back to find a noun that isn't a pronoun. Read pairings of noun and prepositional phrase until you have a sensible phrase.
4. **Be sure you know which noun carries out the action described by each verb**.
5. Using a format easily understood by people regardless of the languages they read and listen to, **draw** a representation of the phrase.
6. **Underline** the phrase.
7. Analyze next phrase.

Table 1. Breaking passage into short phrases

	Phrase	Main features	Sketch
1.	Alice stands	Noun verb	Upright stick figure; capital "A" in face
2.	on a box	Preposition noun	Rectangle immediately under Alice
3.	that has a height	Verb noun	Upward arrow from bottom to top of rectangle
4.	of 2 feet.	Preposition noun	Label upward arrow "2 ft"
5.	The top of Alice's head	Noun preposition noun	Dot marking top of Alice's stick figure
6.	is 5 feet	Verb noun	Label "5 ft" near dot
7.	above the ground.	Preposition noun	Upward arrow from bottom of rectangle to top of Alice's stick figure
8.	How tall is Alice?	Noun verb noun	Upward arrow from bottom to top of Alice's stick figure, labeled "? ft"

Showing the table above is usually unnecessary. Underlining the printed problem statement and drawing a sketch usually suffices.

Alice stands on a box that has a height of 2 feet. The top of Alice's head is 5 feet above the ground. How tall is Alice?



Carry out algebraic reasoning

Table 2. Showing algebraic steps.

Step	Algebraic reasoning
State **question**.	Simplify $3x + 4x$.
Copy **template** from notes or textbook	$ac + bc$ $\downarrow$ $(a + b)c$
Replace template symbols with symbols from current discussion.	$\overset{3}{\cancel{a}}\overset{x}{\cancel{c}} + \overset{4}{\cancel{b}}\overset{x}{\cancel{c}}$ $\downarrow$ $(\overset{3}{\cancel{a}} + \overset{4}{\cancel{b}})\overset{x}{\cancel{c}}$
State **claim** and clean up.	$(3 + 4)x$ $7x$
Finished example	Simplify $3x + 4x$. $\overset{3}{\cancel{a}}\overset{x}{\cancel{c}} + \overset{4}{\cancel{b}}\overset{x}{\cancel{c}}$ $\downarrow$ $(\overset{3}{\cancel{a}} + \overset{4}{\cancel{b}})\overset{x}{\cancel{c}}$ $(3 + 4)x$ $7x$

Narrate algebraic reasoning

Table 3. Showing and describing algebraic steps.

Step	Algebraic reasoning	Narration
State **question**.	Simplify $3x + 4x$.	Simplify $3x + 4x$.
Copy **template** from notes or textbook	$ac + bc$ $\downarrow$ $(a + b)c$	**By** the distributive property, $ac + bc$ can be replaced by $(a + b)c$.
Replace template symbols with symbols from current discussion.	$\overset{3}{\cancel{a}}\overset{x}{\cancel{c}} + \overset{4}{\cancel{b}}\overset{x}{\cancel{c}}$ $\downarrow$ $(\overset{3}{\cancel{a}} + \overset{4}{\cancel{b}})\overset{x}{\cancel{c}}$	**Regard** 3 as a, 4 as b, and x as the common factor c.
State **claim** and clean up.	$(3 + 4)x$ $7x$	**So,** $3x + 4x$ can be replaced by $(3 + 4)x$, in other words, $7x$.
Finished example	Simplify $3x + 4x$. $\overset{3}{\cancel{a}}\overset{x}{\cancel{c}} + \overset{4}{\cancel{b}}\overset{x}{\cancel{c}}$ $\downarrow$ $(\overset{3}{\cancel{a}} + \overset{4}{\cancel{b}})\overset{x}{\cancel{c}}$ $(3 + 4)x$ $7x$	Simplify $3x + 4x$. **By** the distributive property, $ac + bc$ can be replaced by $(a + b)c$. **Regard** 3 as a, 4 as b, and x as the common factor c. **So,** $3x + 4x$ can be replaced by $(3 + 4)x$, in other words, $7x$.

Basic algebra templates

For exercises, see corresponding “EA” sections in Marecek *et al.*, *Elementary Algebra 2e*, available for free through a CC BY license at OpenStax: https://openstax.org/details/books/elementary-algebra-2e

Table 4. Basic algebra templates

EA	Name	Template(s)
1.9	Division by zero is undefined	Any denominator or factor in denominator = 0 $\downarrow$ Discard template
1.5	Product of fractions	$\begin{array}{c} \frac{a}{b}\cdot\frac{c}{d} \\ \updownarrow \\ \frac{ac}{bd} \end{array}$
1.5	Canceling factors in	$\begin{array}{c} \frac{a\cdot c}{b\cdot c} \\ \updownarrow \\ \frac{a}{b} \end{array}$ $\begin{array}{c} \frac{c\cdot a}{c\cdot b} \\ \updownarrow \\ \frac{a}{b} \end{array}$
1.5	Quotient of fractions	$\begin{array}{c} \frac{\left(\frac{a}{b}\right)}{\left(\frac{c}{d}\right)} \\ \updownarrow \\ \frac{a}{b}\cdot\frac{d}{c} \end{array}$
1.9	Commutative Property of Addition	$\begin{array}{c} a+b \\ \updownarrow \\ b+a \end{array}$
1.9	Commutative Property of Multiplication	$\begin{array}{c} a\cdot b \\ \updownarrow \\ b\cdot a \end{array}$
1.9	Associative Property of Addition	$\begin{array}{c} (a+b)+c \\ \updownarrow \\ a+(b+c) \end{array}$
1.9	Associative Property of Multiplication	$\begin{array}{c} (a\cdot b)\cdot c \\ \updownarrow \\ a\cdot(b\cdot c) \end{array}$
1.9	Distributive Property	$\begin{array}{c} a\cdot(b+c) \\ \updownarrow \\ a\cdot b+a\cdot c \end{array}$ $\begin{array}{c} (b+c)\cdot a \\ \updownarrow \\ b\cdot a+c\cdot a \end{array}$ $\begin{array}{c} a\cdot(b-c) \\ \updownarrow \\ a\cdot b-a\cdot c \end{array}$ $\begin{array}{c} (b-c)\cdot a \\ \updownarrow \\ b\cdot a-c\cdot a \end{array}$ $\begin{array}{c} (a+b)\cdot(c+d) \\ \updownarrow \\ \begin{array}{c\|c\|c} & c & d \\ \hline a & a\cdot c & a\cdot d \\ \hline b & b\cdot c & b\cdot d \end{array} \\ a\cdot c+b\cdot c+a\cdot d+b\cdot d \end{array}$
1.9	Identity Property of Addition	$\begin{array}{c} a+0 \\ \updownarrow \\ a \end{array}$ $\begin{array}{c} 0+a \\ \updownarrow \\ a \end{array}$
1.9	Identity Property of Multiplication	$\begin{array}{c} a\cdot 1 \\ \updownarrow \\ a \end{array}$ $\begin{array}{c} 1\cdot a \\ \updownarrow \\ a \end{array}$
1.9	Identity Property of Division	$\begin{array}{c} \frac{a}{1} \\ \updownarrow \\ a \end{array}$
1.9	Adding additive inverse is equivalent to subtracting	$\begin{array}{c} a+(-b) \\ \updownarrow \\ a-b \end{array}$
1.9	Inverse Property of Addition	$\begin{array}{c} a+(-a) \\ \updownarrow \\ 0 \end{array}$ $\begin{array}{c} a-a \\ \updownarrow \\ 0 \end{array}$
1.5 & 1.9	Inverse Property of Multiplication (and the related One Property of Division)	$\begin{array}{c} a\cdot\frac{1}{a} \\ \updownarrow \\ 1 \end{array}$ $\begin{array}{c} \frac{a}{a} \\ \updownarrow \\ 1 \end{array}$
1.9	Multiplication by zero	$\begin{array}{c} a\cdot 0 \\ \updownarrow \\ 0 \end{array}$ $\begin{array}{c} 0\cdot a \\ \updownarrow \\ 0 \end{array}$
1.9	Division involving zero	$\begin{array}{c} \frac{0}{a} \\ \updownarrow \\ 0 \end{array}$ See “Division by zero is undefined”
2.1	Addition Property of Equality	$\begin{array}{c} a=b \\ \downarrow \\ \begin{array}{ccc} a & = & b \\ +c & & +c \\ \hline a+c & = & b+c \end{array} \end{array}$
2.1	Subtraction Property of Equality	$\begin{array}{c} a=b \\ \downarrow \\ \begin{array}{ccc} a & = & b \\ -c & & -c \\ \hline a-c & = & b-c \end{array} \end{array}$
2.2	Multiplication Property of Equality	$\begin{array}{c} a=b \\ \downarrow \\ a\cdot c=b\cdot c \end{array}$
2.2	Division Property of Equality	$\begin{array}{c} a=b \\ \downarrow \\ \frac{a}{c}=\frac{b}{c} \end{array}$
6.2	Exponent (special case of squaring)	$\begin{array}{c} a^2 \\ \updownarrow \\ a\cdot a \end{array}$
7.6	Zero-Product Property	$\begin{array}{c} a\cdot b=0 \\ \downarrow \\ a=0 \text{ or } b=0 \end{array}$
10.3	Quadratic Formula	$\begin{array}{c} ax^2+bx+c=0 \\ \updownarrow \\ x=\frac{-b\pm\sqrt{b^2-4ac}}{2a} \end{array}$

Determine logical relationship between two passages

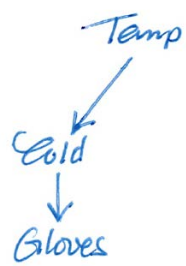
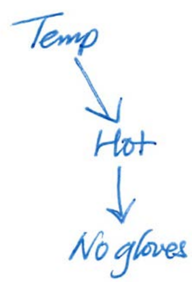
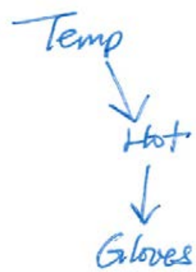
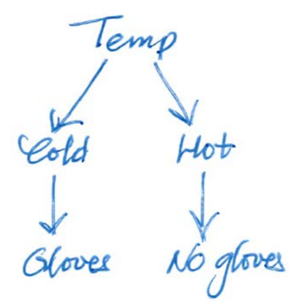
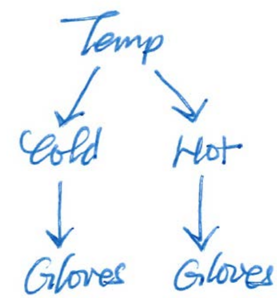
Are passages 1 and 2 consistent? If so, can passage 2 be inferred from passage 1?

Passage 1: Denise wears gloves when it's cold.

Passage 2: Denise doesn't wear gloves when it's hot.

Solution: Fill out Table 5 below.

Table 5. Diagram passages to determine how they relate

<table>
<tr><th></th><th>Passage 1</th><th>Passage 2</th><th>Alternative to passage 2</th><th>Propose World A described by passage 1 and passage 2.</th><th>Propose World B described by passage 1 and alternative to passage 2.</th></tr>
<tr><td>Text</td><td>Denise wears gloves when it's cold.</td><td>Denise doesn't wear gloves when it's hot.</td><td>Denise wears gloves when it's hot.</td><td>Denise wears gloves when it's cold. Denise doesn't wear gloves when it's hot.</td><td>Denise wears gloves when it's cold. Denise wears gloves when it's hot.</td></tr>
<tr><td>Diagram</td><td>
</td><td>
</td><td>
</td><td>
</td><td>
</td></tr>
<tr><td>Possible?</td><td>Possible</td><td>Possible</td><td>Possible</td><td>Possible</td><td>Possible</td></tr>
</table>

In the Table 6 below, look for a row that matches the bottom row of Table 5.

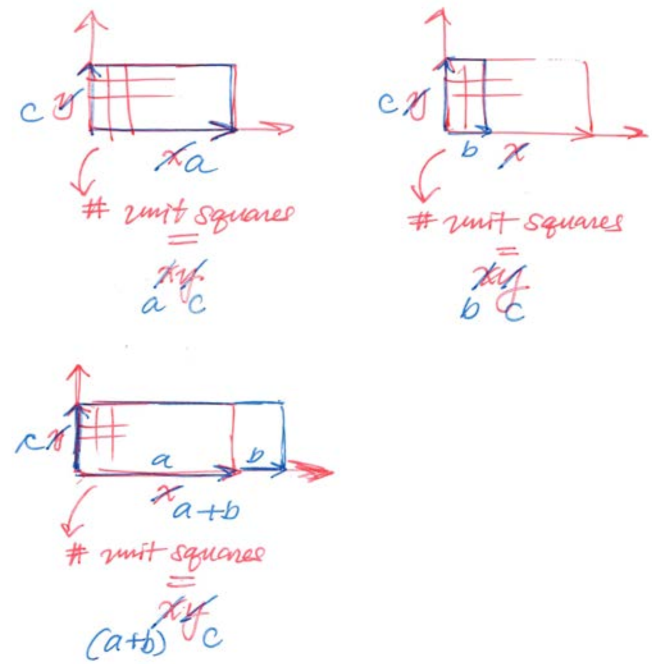
Table 6. Conclusions corresponding to whether different passages and combinations of passages are possible.

Possible	Possible	Possible	Possible	Possible	Passage 2 is **consistent with**, but **cannot be inferred from**, passage 1.
Possible	Possible	Possible	Possible	Impossible	Passage 2 is **consistent with** and **can be inferred from** passage 1.
Possible	Possible	Possible	Impossible	Possible	Passage 2 is **inconsistent** with passage 1.

Answer: Passage 2 is **consistent with**, but **cannot be inferred from**, passage 1.

“Derive” an algebraic rule from a graphical representation

Table 7. Using multiple representations when justifying an algebraic rule

Narration of template	**Algebraic template**	**Graphical justification**
By the distributive property, $ac + bc$ can be replaced by $(a + b)c$.	$ac + bc$ $\downarrow$ $(a + b)c$	

Multiplication of real numbers is graphically represented by drawing a 2-dimensional coordinate system, with a horizontal arrow from the origin along the horizontal axis representing the real number x and a vertical arrow from the origin along the vertical axis representing the real number y. A rectangular region with the x- and y- arrows as two sides is partitioned into unit squares. A caption emanating from the rectangular region is labeled number of unit squares = xy. This graphical template is drawn three times. In the first, the letters x and y are slashed out and replaced by, respectively, a and c. In the second, the letters x and y are slashed out and replaced by, respectively, b and c. In the third, the letters x and y are slashed out and replaced by, respectively, $a + b$ and c. The horizontal arrow labeled $a + b$ is broken into two pieces, labeled a and b. The horizontal arrows labeled a are congruent, and the horizontal arrows labeled b are congruent. The number of unit squares representing the product ac plus the number of unit squares representing the product bc equals the number of unit squares representing the product $(a + b)c$, so $ac + bc$ can be replaced by $(a + b)c$.