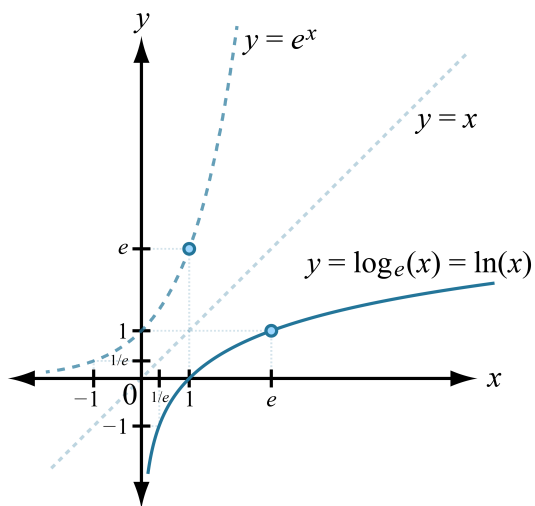


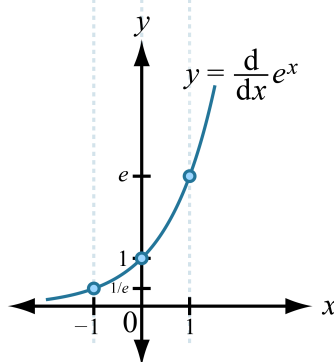
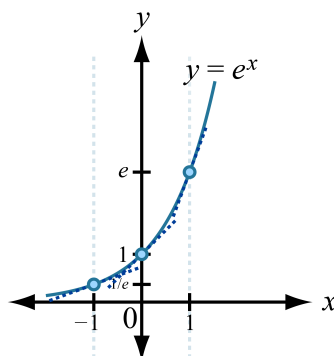
Calculus of exponential and logarithmic functions



Change-of-base identities are useful for calculating derivatives and integrals of exponential and logarithmic functions in terms of other bases:

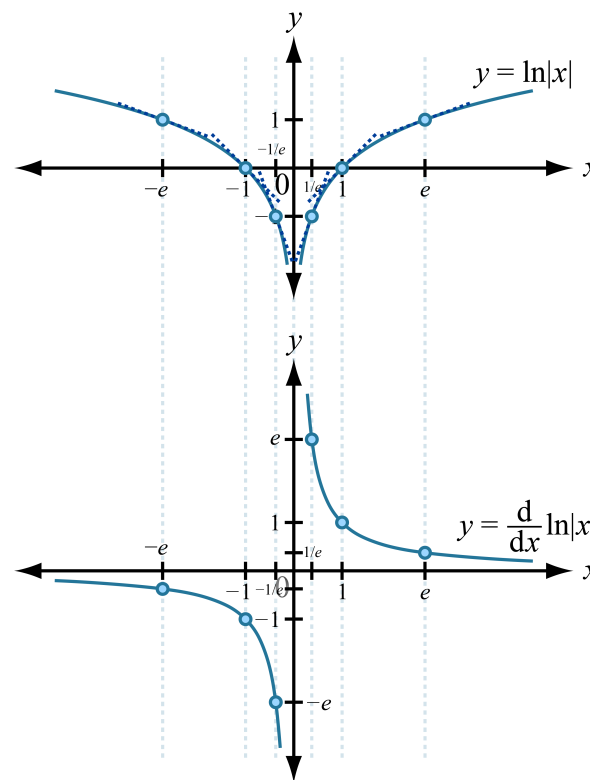
$$b^x = (e^{\ln b})^x$$

$$\log_b x = \frac{\ln x}{\ln b}$$



$$\frac{d}{dx} e^x = e^x$$

$$\int e^x dx = e^x + C$$



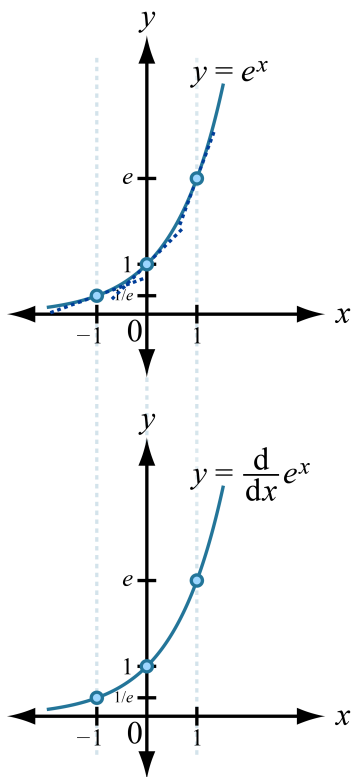
$$\frac{d}{dx} \ln|x| = \frac{1}{x}$$

$$\frac{d}{dx} \ln x = \frac{1}{x}, \quad x > 0$$

$$\int \frac{1}{x} dx = \ln|x| + C$$

Calculus of exponential and logarithmic functions

Appendix 1: Symbolic derivation of derivative of exponential function



$$\frac{d}{dx} b^x = \lim_{\Delta x \rightarrow 0} \frac{b^{x+\Delta x} - b^x}{\Delta x}$$

$$= \lim_{\Delta x \rightarrow 0} \frac{b^x b^{\Delta x} - b^x}{\Delta x}$$

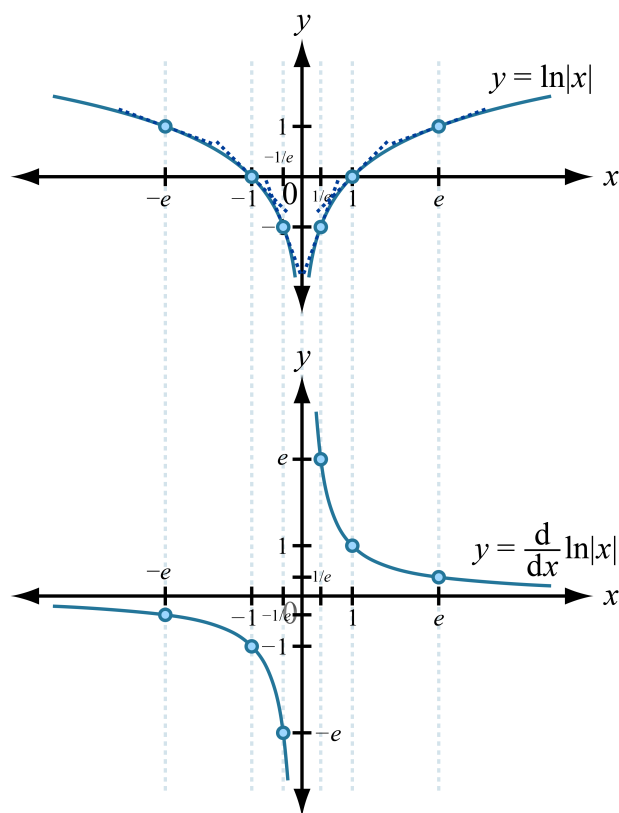
$$\frac{d}{dx} b^x = b^x \underbrace{\left(\lim_{\Delta x \rightarrow 0} \frac{b^{\Delta x} - 1}{\Delta x} \right)}_{\substack{\text{if quantity} = 1, \\ \text{rename } b \text{ as } e}}$$

$$\frac{d}{dx} e^x = e^x$$

$$\int e^x dx = e^x + C$$

Calculus of exponential and logarithmic functions

Appendix 2: Symbolic derivation of derivative of natural logarithm of absolute value function



$\ln(x)$ is the inverse of e^x

$$g'(x) = \frac{1}{f'(g(x))}$$

$$\begin{aligned} \frac{d}{dx} \ln x &= \frac{1}{e^{\ln x}} \\ &= \frac{1}{x}, \quad x > 0 \end{aligned}$$

$$\begin{aligned} \frac{d}{dx} \ln|x| &= \frac{d}{dx} \ln \sqrt{x^2} \\ &= \frac{1}{e^{\ln \sqrt{x^2}}} \cdot \frac{1}{2} (x^2)^{-\frac{1}{2}} \cdot 2x \\ &= \frac{1}{|x|} \cdot \frac{1}{|x|} \cdot x \end{aligned}$$

$$\frac{d}{dx} \ln x = \frac{1}{x}, \quad x > 0$$

$$\frac{d}{dx} \ln|x| = \frac{1}{x}$$

$$\int \frac{1}{x} dx = \ln|x| + C$$