

# Partial fractions and introduction to logistic growth

1. Solve

$$\int \frac{2x + 3}{x^2 + 3x + 2} dx$$

Use  $u$ -substitution:

$$u = x^2 + 3x + 2$$

$$du = (2x + 3)dx$$

$$\int \frac{2x + 3}{x^2 + 3x + 2} dx = \int \frac{1}{u} du$$

$$= \ln|u| + C$$

$$= \ln|x^2 + 3x + 2| + C$$

2. Solve

$$\int \frac{1}{x^2 + 3x + 2} dx$$

$u$ -substitution will not work

$$\frac{1}{x^2 + 3x + 2} = \frac{1}{(x + 2)(x + 1)}$$

$$= \frac{A}{x + 2} + \frac{B}{x + 1}$$

$$= \frac{A(x + 1)}{(x + 2)(x + 1)} + \frac{B(x + 2)}{(x + 2)(x + 1)}$$

$$\Rightarrow 1 = A(x + 1) + B(x + 2)$$

Let  $x = -1$

$$1 = A((-1) + 1) + B((-1) + 2)$$

$$1 = A(0) + B(1)$$

$$1 = B$$

Let  $x = -2$

$$1 = A((-2) + 1) + B((-2) + 2)$$

$$1 = A(-1) + B(0)$$

$$-1 = A$$

$$\frac{1}{x^2 + 3x + 2} = \frac{-1}{x + 2} + \frac{1}{x + 1}$$

$$\int \frac{1}{x^2 + 3x + 2} dx = \int \left( \frac{-1}{x + 2} + \frac{1}{x + 1} \right) dx$$

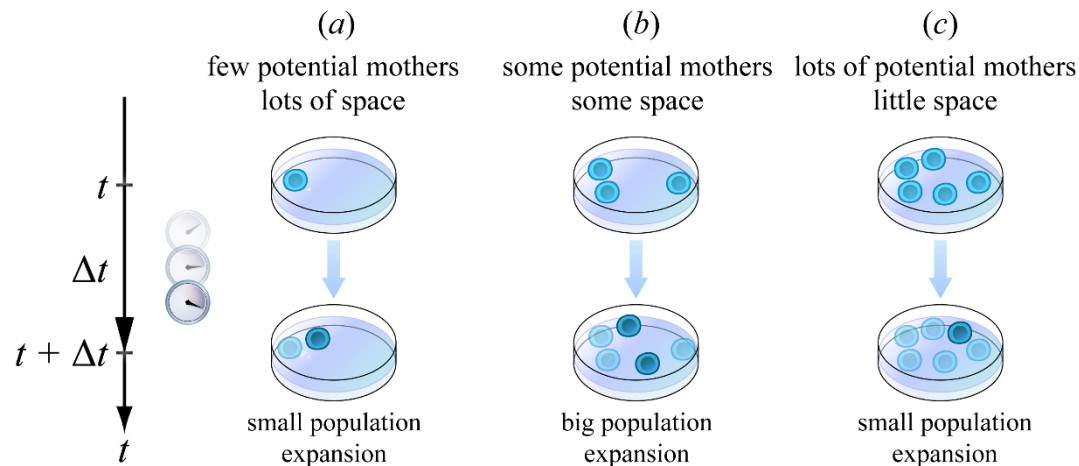
$$= \int \frac{-1}{x + 2} dx + \int \frac{1}{x + 1} dx$$

$$= -\ln|x + 2| + \ln|x + 1| + C$$

$$= \ln \frac{|x + 1|}{|x + 2|} + C$$

Ensure degree of numerator is less than degree of denominator before using this method (can use polynomial long division).

## Logistic growth



$P$  – **population** (number of individuals)

$K$  – **carrying capacity** (number of individuals at which there is “no more space”)

$$\Delta P \approx \frac{r}{K} P(K - P)\Delta t$$

$$\frac{dP}{dt} = rP \frac{(K - P)}{K}$$

**Exercise:** Solve for  $P(t)$  given  $P(0)$ . Find  $\lim_{t \rightarrow \infty} P(t)$  with and without using solution.