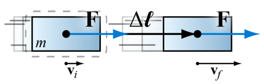
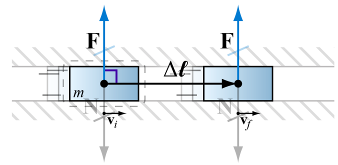
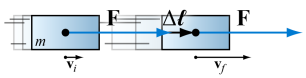
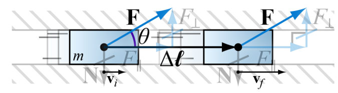
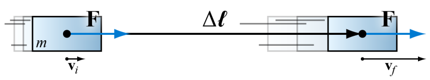
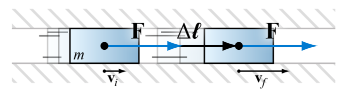
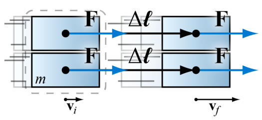
A net force can perform work that changes the $\frac{1}{2}mv^2$ of an object

How much can I change the v^2 of an object of mass m by applying a constant force while the object moves through a path length?

	$\Delta(v^2) \neq 0$		$\vec{\mathbf{F}} \perp \Delta\vec{\boldsymbol{\ell}} \Rightarrow \Delta(v^2) = 0$
	$\uparrow \lvert\vec{\mathbf{F}}\rvert \Rightarrow \uparrow \lvert\Delta(v^2)\rvert$		oblique $\Rightarrow$ some $\lvert\Delta(v^2)\rvert$
	$\uparrow \Delta\ell \Rightarrow \uparrow \lvert\Delta(v^2)\rvert$		$\vec{\mathbf{F}} \parallel \Delta\vec{\boldsymbol{\ell}} \Rightarrow \max \lvert\Delta(v^2)\rvert$
	$\uparrow \lvert\sum\vec{\mathbf{F}}\rvert \Leftarrow \uparrow m$		

Deduced **relationship**

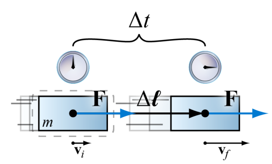
$$\underbrace{\left(\sum F\cos\theta\right)}_{\Sigma F_{\parallel}}\Delta\ell = \frac{1}{2}m\Delta(v^2)$$

$$= \frac{1}{2}m\left(v_f^2 - v_i^2\right)$$

$$= \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2$$

$$= \Delta\left(\frac{1}{2}mv^2\right)$$

Vocabulary



Kinetic energy

$$K := \frac{1}{2}mv^2$$

Work delivered by a force

$$\Delta W_{\mathrm{F}} := \underbrace{(F\cos\theta)_{\mathrm{AVG}}}_{F_{\parallel,\mathrm{AVG}}}\Delta\ell$$

Power delivered by a force

$$P_{\mathrm{F,AVG}} := \frac{\Delta W_{\mathrm{F}}}{\Delta t}$$

For AP Physics C,

$$P_{\mathrm{F}} := \frac{\mathrm{d}W_{\mathrm{F}}}{\mathrm{d}t}$$

Work-energy theorem

$$K_i + \sum_{\mathrm{F}} \Delta W_{\mathrm{F}} = K_f$$

For a system having no internal degrees of freedom (e.g. idealized particle),

$$\sum_{\mathrm{F}} \Delta W_{\mathrm{F}} = \Delta W_{\Sigma\vec{\mathbf{F}}}$$

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Work done by a varying force

Consider the work performed by a force of varying strength. Allow increments of path length to be small enough so that, for each increment, the force is roughly constant.

$$\Delta W_{\mathrm{F},k} \approx F_{\parallel,k}\Delta\ell$$

The total work done along a path of finite length

$$\Delta W_{\mathrm{F}} \approx \sum_k F_{\parallel,k}\Delta\ell$$

is the signed area "under" the plot of $F_\parallel$ vs. ℓ.

For AP Physics C,

$$\Delta W_{\mathrm{F}} = \int_{\ell=\ell_i}^{\ell=\ell_f} F_\parallel \,\mathrm{d}\ell$$